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THE FACULTY OF BUSINESS MANAGEMENT

ELABORATION OF THE THEORETICAL FOUNDATIONS OF THE INTEGRATED MARKET DEMAND

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I confirm that the work is presented in the format defined by the International Black Sea University (IBSU) and that it is relevant to the field of study. I hereby confirm the practical and scientific value of the thesis and that it contributes significantly towards improving economic knowledge, strategies and policy issues in Georgia.

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ABSTRACT

Forecasting of the sales volume and planning of the production process, as well as other crucial managerial decisions are based on the knowledge of the demand for the good and service under consideration. The relationship between the price and the quantity demanded is obvious. Therefore, for the successful business decisions, it is important to understand the impact of pricing on sales by estimating the demand function for the product.

There are two main approaches to this issue: one is based on the traditional and modern theories of consumer behavior; whereas the second is a pragmatic approach to demand analysis. The latter omits the theoretical frames of the fundamental law of demand, and formulates demand functions directly on the basis of observed market data.

The traditional and modern concepts of consumer behavior are theoretically impressive and useful for the academic and in-class simulation of the demand-price relationship; however, it has no practical value for managers of the business firms in dealing with complexity of the real world. Mostly, this approach employs the linear relationship of the demand and price in outlining the fundamentals of the theories.

The pragmatic approach is based on functional forms of the demand analysis. They are useful in making managerial business decisions. However, they are difficult to estimate, requiring much computational power of the analyst as well as specific statistical software. Mostly, this approach takes into account nonlinear forms of the demand and price relationships: distributed lag models, indirect translog model, and etc.

The true total revenue maximization requires estimation of market demand curve on the entire domain. It is well known that almost in all cases the entire market demand curve is of essentially nonlinear form. The latter implies the fact, that it is impossible to estimate optimal parameters of total revenue maximization by means of local linearization. Linearization of the demand curve can be applied locally, but not to the whole domain of market demand curve (globally).

The described situation predetermines the main objective of the present research: elaborating theoretical foundations and a mathematical model of essentially non-linear demand-price relationship, which permits easy and reliable estimation of all of its parameters, and eventually, reliable estimation of optimal revenue and profit.

In the present research we suggest a new method of analysis of demand's internal structure and compound nature, dependent on the contributions of various groups of customers. New concepts of Observed Demand D, Smoothed Demand D_s, Elementary Observed Demand D_i, Smoothed Elementary Demands D_{si}, and Integrated Demand $D_{A\Sigma}$ are introduced. Direct and inverse problems of estimation of elementary and integrated market demand's parameters are defined. Integrated market demand structure is represented as a multidimensional dummy variables regression model. The theoretical results are verified by means of corresponding numerical examples.

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INTRODUCTION

Demand estimation problem arises each time when there is need for forecasting of the sales volume, optimal price settlement for profit maximization, or for empirical studies of the market for demand. There is a relationship between price and quantity demanded and it is important to understand the impact of pricing on sales by estimating the demand curve for the product. Market demand for the product can be obtained by survey, and observations; or experiments can be performed at prices above and below the current price in order to determine the price elasticity of demand.

One should be aware of the difference between demand estimation and demand forecasting. The estimation is an attempt to quantify the links between the demand and the parameters which determine it. The forecasting is to attempt to predict the overall future demand.

Businesses, that want to know what level demand is likely to be in future, will use a forecasting technique, while businesses that want to know how the pricing policy could be used to obtain a given increase in demand would use an estimation technique.

Businesses need to have information about possible future demand in order to pursue optimal pricing strategy. They can only set a price that the market will meet if they are to sell the product. Therefore, optimistic estimates of demand may lead to quite a high price and decrease of sales. Vice versa, pessimistic estimates of demand may lead to a price which is set too low and causes loss of profits. The more accurate information the businesses have the lower the probability to make a decision which may have a negative impact on operations and profitability.

Demand estimation is a process that involves coming up with an estimate of the amount of demand for a product or service. The estimate of demand is typically framed to a particular period of time. Obviously, this is not a method to forecast the future for the business; under correct assumptions it is meant to be used to deliver accurate estimates.

One of the reasons that businesses use demand estimation is to assist with pricing. In offering a new product or starting a new business, it is difficult to set the price. Thus, if there is an estimation of what the demand will be for the product, then it is much easier to come up with the certain price range to charge.

Another reason for the demand estimation is to help with production. Before a business undertakes production, it should have an estimate of the demand for that product. The objective is not to over or under produce the product and to avoid having invested into inventory that may not even be needed if the demand is low.

It is important to remember when making decisions based on demand estimations, that these estimations are just educated guesses about what will be the demand for a product or service. Always there should be some space for error in the estimation of the demand for the business.

The current study focuses on the alternative approach to demand estimation analysis based on the concept of horizontal summation of elementary demand observations, as well as provides an alternative approach to calculation of optimal parameters of the revenue maximization.

The Problem Actuality: The demand estimation for optimal decisions is problem of the utmost importance for any business entity willing to achieve success and to continue operations in the selected market niche. The estimation starts with data observations for the existing goods and services and with experimental focus group of customers for the new developed goods and services. Later, collected data is analyzed through applications of different sophisticated regression methods to reveal the relationship between the price and quantity of the product under consideration. Based on the results obtained from regression analysis the forthcoming demand for the product, the potential revenue, the potential customer groups, the operational strategy of the firm, and other important decisions are defined. The real reflection of the market condition for the product will solely depend on the accuracy of data observations conducted and parameters set forward by the researcher. Therefore, in order to have healthy strategy and success in the market, the demand structure should be estimated near to the real market values. However, difficulty in estimating demand curve lies in the decision of which regression method to apply: linear, logarithmic, polynomial, moving averages, and so on. Each method will require the researcher to decide what depth of analytical complexity to choose and later to determine if observed data are explained through the resulting relationship. There is no ready to apply solution. Each of the conventional methods is based on the complicated statistical methods and requires analytical abilities and experience of the researcher.

The current research is addressing to the complexity of the estimation problem of the demand function by proposing alternative approach by assumption that market demand is a set of linear demand-price components that recovers entire market demand.

The Subject of the Research: The subjects of the research are the concepts of micro- and managerial economics: demand-price relationships, optimal demand and price, total revenue etc. The research employs the econometrical methods of multidimensional regression analysis and computer programming.

The Aim of the Research: The general purposes of this research are: 1. elaboration of new methods of analysis and estimation of non-linear integrated demand-price relationships and 2. elaboration of new methods of estimation and computation of micro-economical optimal parameters (total revenue curve, optimal values of price, demand quantity and total revenue) for non-linear integrated demand-price relationship.

The following objectives have been identified:

- Representation of an observed demand-price relationships as the integrated essentially non-linear object consisting of a set of linear demand-price components;
- Elaboration of the discrete and continuous theoretical models of integrated non-linear demand-price processes;
- Elaboration of the new statistical method of estimation of parameters for integrated demand discrete model. Testing of the method throughout several numerical examples;
- Comparison of the integrated demand discrete model results to the conventional polynomial regression analysis methodology;
- Creation of the appropriate software tools;
- Based on developed integrated demand discrete model, elaboration of the new approach to the estimation of optimal microeconomic parameters;
- Elaboration of the new methodology of defining of maximum of integrated total revenue curves;
- Elaboration of the new expressions for the estimation of the optimal values of prices, demand quantities and maximums of total revenues.

Hypothesis: The market demand curve is an integration of a set of the linear demand-price relationship components representing demands of a different customer groups, distinguished by social status, location of residence, and so on; which further can be used to elaborate analysis of microeconomic parameters to obtain optimal values of prices, demand quantities, and maximums of total revenues.

The Methodology of Research: The methodology of the research is based on principals and methods of micro-economical and managerial economic analysis; methods of calculus, linear algebra and mathematical statistics, in particular multidimensional dummy variable regression analysis.

The Novelty and Contributions:

1. The observed demand was represented as the integrated, essentially non-linear, object consisting of a set of linear demand-price components;
2. On the base of the new introduced concepts of *Observed Demand, Smoothed Demand, Elementary Observed Demand, Smoothed Elementary Demands, Integrated Demand, and Integrated Total Revenue Curve*, the discrete and continuous theoretical models of integrated demand were elaborated;
3. Based on the usage of the dummy multidimensional regression technique, the new statistical method of estimation of parameters for integrated demand discrete model was elaborated. The capability of the model was demonstrated throughout several numerical examples;
4. The new integrated demand discrete model results were compared to the conventional polynomial regression analysis results. It was shown that the integrated demand discrete model gives statistically better results in explaining the observational data;
5. Appropriate software tools (in MATLAB programming language) were created;
6. Based on elaborated integrated demand discrete model the new approach to the estimation of optimal microeconomic parameters was elaborated;
7. The new methodology of determining of maximum of integrated total revenue curves was elaborated;
8. Taking into consideration nonlinear specific nature of integrated demand, the new expressions for estimation of optimal values of prices, demand quantities and maximums of total revenues were obtained;

9. The new conditions and relevant criteria for existing points of maxima of the integrated total revenue curve were defined;

10. The outcomes mentioned above allowed creating the new principles of analysis and calculation of optimal micro-economical parameters for nonlinear integrated demand-price functional relationships.

Theoretical Value and Practical Importance of the Study: The current research is a new step in managerial economics; it will contribute to the ongoing developments in measuring demand by elaborating theoretical foundations and a mathematical model of essentially non-linear demand-price relationship which permits easy and reliable estimation of all its parameters and eventually, reliable estimation of optimal revenue and profit. The developed model is applicable to determine the entire market demand from observed linear components, or to determine linear components from the observed entire market demand.

The research can be of particular importance as an alternative instrument in the analysis of demand process for revenue and profit maximization strategies by business entities, researcher, scholars, practitioners, and so on.

The main findings and conclusions of the study have been presented on the various conferences and described in 4 articles published in national and international, refereed and peer reviewed, academic journals.

CHAPTER 1. LITERATURE REVIEW

All business firms would like to know the current as well as the likely demand for their goods and services. Particularly, firms would like to know how much of a given product they would sell in a given market in a given period; whether the sales would increase or decrease from the current levels and by how much; and what would be the share of the market. This information is very critical for a firm. It cannot arrange any of its activities without this knowledge. This crucial data is obtained by measuring the demand and forecasting the sales.

The product market is composed by demand for goods and services from one side, and by supply of goods and services from another side. It is obvious that if there is no demand for a good and service then there is no need to produce and offer that good and service. In the same logic, if there is shortage in supply for a good and service, then it may be required to increase its production. Moreover, process of producing takes a time, so that producer may be willing to know possible future demand quantity for a relevant good and service in order to plan its production accordingly. Thus, a clear vision of the appropriate demand quantity is one of the key indicators of the success of any business firm.

As indicated by the studies of Ben-Akiva and Lerman (1985), Li and Azarm (2000), Cook (1997), Donndelinger and Cook (1997), demand analysis aims to identify and analyze the factors that influence the demand. A business firm is not a passive taker of the demand; it has the capacity to create demand for its product as well. Thus, according to Gupta (2001), a study of demand is necessary for a decision-maker, for it has bearings on its production agenda, and influence on its profit, among other critical variables, it is also subject to handling by the decision-maker and it is crucial for reaching the firm's objectives.

1.1. Importance of Demand Analysis in Business Decisions

The demand analysis and the demand theory are of utmost importance to the business firms. The one of the useful information for business decision making is obtained through them. Gupta (2001) notes that success or failure of business firms depend mainly on their ability to generate resources by satisfying the demand of consumers. The firms that fail the task of attracting consumers to their products are soon forced to quit the market.

The significance of demand analysis in business decision processes is emphasized in studies of Hirschey (2006), McGuigan et al (2010), Gylfason (1999), Froeb and McCann (2009), Spulber (2009), Gupta (2001), Varian (2010) and can be summarized under following titles:

- Sales forecasting;
- Pricing Decisions;
- Marketing decisions;
- Production Decisions
- Financial Decisions.

1.1.1. Demand analysis for sales forecasting

The firm's production is dependent on sales, which is based on the demand for the product of the firm. Therefore, prediction of the volume of the sales can be performed on the basis of the demand.

Moon and Mentzer (1999) noted that any sales forecast should be thought of as a best guess about customer demand for a company's goods or services, during a particular time horizon, given a set of assumptions about the environment.

According to Anthony and Govindarajan (2007) a forecast works as a management control. A forecast, by Merchant and Van der Stede (2007), is, when correctly used, a cost-reducer, but besides that, it also works as a motivational, coordinating and controlling tool for people involved in the process.

Armstrong (2003) means that forecasting methods can be used by planners to predict outcomes from alternative plans. Although forecasts can be employed for plans, research emphasizes that forecasting only is useful when the techniques and methods handled are applied to an organization's decision-making and planning processes. It is emphasized that a strong bridge between the theories and the practical use in an organization is required for an efficient use in management situations. (Winklhofer, Diamantopoulos, and Witt, 1996)

The forecast of sales volume creates a believed future sales volume, which sets indicators to the production, the logistical, the purchasing, the marketing and the financial function of an organization. For example, if demand quantity is excessive, sales volume will be excessive and if demand quantity is little, sales volume will be little. Therefore, sales forecast affects the firm's arrangement decisions on whether to increase or reduce production or push up sales.

1.1.2. Demand analysis for pricing decisions

Demand-based pricing is any pricing method that uses consumer as the central element. These include, summarized by Nagle and Hogan (2005), price skimming, price discrimination, psychological pricing, penetration pricing, value-based pricing.

Price skimming is a pricing strategy in which a marketer sets a relatively high price for a product or service at first, and then lowers the price over time. In other words, price skimming is when a firm charges the highest initial price that customers will pay. As the demand of the first customers is satisfied, the firm lowers the price to attract another, more price-sensitive segment. The objective of a price skimming strategy is to capture the consumer surplus. It allows the firm to recover its sunk costs quickly before competition steps in and lowers the market price. If this is done successfully, then theoretically no customer will pay less for the product than the maximum they are willing to pay.

Price discrimination exists when sales of identical goods or services are transacted at different prices from the same provider. Price discrimination requires market segmentation and some means to discourage discount customers from becoming resellers and, by extension, competitors. This usually entails using one or more means of preventing any resale, keeping the different price groups separate, making price comparisons difficult, or restricting pricing information.

Psychological pricing is a marketing practice based on the theory that certain prices have a psychological impact. The retail prices are often expressed as “odd prices”: a little less than a round number, e.g. $19.99. The theory is that this pricing drives demand greater than would be expected if consumers were perfectly rational.

Penetration pricing is the pricing technique of setting a relatively low initial entry price, often lower than the eventual market price, to attract new customers. Penetration pricing is most commonly associated with a marketing objective of increasing market share or sales volume, rather than to make profit in the short term.

Value-based pricing sets prices primarily, but not exclusively, on the value, perceived or estimated, to the customer rather than on the cost of the product or historical prices. Value-based-pricing is most successful when products are sold based on emotions, in niche markets, in shortages, or for indispensable add-ons. By definition of Nagle and Hogan (2005), long term prices based on value-based pricing are always higher or equal to the prices derived from cost-based pricing.

1.1.3. Demand analysis for marketing decisions

Demand analysis plays a more central role in marketing than perhaps any other field in the social sciences. Chintagunta and Nair (2011) in their study note that normatively, models of demand are used to forecast the effect of marketing interventions, and to prescribe the implementation of better policies that increase the profits of firms or improve the welfare of consumers. Positively, models of demand are used to test theories of consumer response and to quantify the effects of marketing in competitive environments.

Summarizing Hughes (1973), the study of demand helps a firm to formulate marketing decisions, so that it analyses and measures the forces that determine the demand. The demand can be influenced by manipulating the factors on which consumers base their demand on goods and services.

According to Kotler (1973) the task of marketing management is not simply to build demand but rather to regulate the level, timing, and character of demand for the organization's products in terms of its objectives at the time. This view applies to all organizations.

Kotler (1973) differentiates demand states from view point of marketing process and proposes strategies for each case as follows:

- *Negative Demand*

  The market is in a state of negative demand if a major part of the market dislikes the product and may even pay a price to avoid it. The marketing task is to analyze, why the market dislikes the products?

- *No Demands*

  Target consumers may be uninterested in the product. The marketing task is to find ways to connect the benefits of the products to the person's natural needs and interests.

- *Latent Demand*

  Many consumers may share a strong need that cannot be satisfied by any existing products. The marketing task is to measure the size of the potential market and develop effective goods and services that would satisfy the demand.

- *Declining Demand*

  A substantial drop in the demand for products. The marketing task is to:

  - Analyze the cause of market decline.

- Determine whether the demand can be re-stimulated by changing target markets, changing product features and developing more effective goods.
- To reverse the declining demand through creative remarketing of the product.

- *Irregular Demand*

  Organizations face demand that varies on a seasonal, daily or even hourly basis, causing problems of idle capacity or overcrowded capacity. The marketing task is to alter pricing, promotion and other incentives.

- *Full Demand*

  Organizations face full demand when they are pleased with their volume of business. The marketing task is to:

  - Maintain the current level of demand in the face of changing consumer preferences and increasing competition.
  - Quality should be improved.
  - Continuously measure consumer satisfaction.

- *Overfull Demands*

  Some organizations face a demand level that is higher than they can or want to handle. Marketing task is de-marketing which requires finding ways to reduce the demand temporarily or permanently.

- *Unwholesome Demand*

  Unwholesome products will attract organized effort to discourage their consumption. Un-selling campaigns have been conducted against cigarettes, alcohols, hard drugs, handguns and pirated movies.

1.1.4. Demand analysis for production decisions

How much a firm can produce depends on its capacity. But how much it should produce depends on demand. Production is not necessary if there is no demand. But continuous production schedule is necessary if the demand for the production is relatively stable. If the demand is less than the quantity of production, new demand should be created by means of promotional activities such as advertising.

An accuracy of the demand forecast significantly affects safety stock and inventory levels, inventory holding costs, and customer service levels. The demand forecast is one among several critical inputs of a production planning process. When the forecast is inaccurate, the obtained production plan will be unreliable, and may result in over- or under-stock problems. To avoid them, a suitable amount of safety stock must be provided, which requires additional investment in inventory and results in an increased inventory holding costs. (Yenradee et al., 2007).

1.1.5. Demand analysis for financial decisions

The demand condition in the marker for firm's product's affects the financial decisions as well. If the demand for firm's product is strong and growing, the needs for additional finance will be greater. Hence, the financial manager should make necessary financial arrangement to finance the growing need of the capital.

One effect is that higher and more inelastic demand reduces the expected costs of financial distress, as firms are less likely to face low states of cash flow realization. Firms with strong demand are also less prone to predatory behavior by competitors. As a result, strong consumer demand enhances the debt capacity of a firm. In addition, a firm with high and inelastic demand enjoys larger and more certain future cash flow and, therefore, can hold less cash for precautionary reasons and distribute more to its shareholders. (Larkin, 2010)

1.2. Meaning of Demand

In economics, the meaning of demand, as defined by Gupta (2001) is the effective demand, when all its three core characteristics are satisfied – wish to have a good, willingness to pay for that good, and ability to pay for that good. It said to be no demand if any of these characteristics are missing. For example, a vegetarian person may be characterized as having both the willingness to pay as well as the ability to pay for a hamburger; however he does not have a demand for it. This is because he does not desire to have a product and sub-products of any animal meat. Similarly, a businessman might have the wish to have a mobile phone Nokia "Lumia", he might be rich enough to be able to pay for it, but if he is not willing to pay for the mobile phone, he does not have a demand for this product. Also, a university student may be characterized as having both the wish for a scooter as well as the willingness to pay for it, but if he does not have enough money to pay for it, he does not have a demand for the scooter. In contrast to these three situations, a professor, who has the wish for a car, as well as both the will and ability to pay for it, has the demand for a car. Incidentally, sometimes there is a shortage of a commodity, meaning that there is no one from whom the product can be purchased. For example, a professor's demand might be for a new Hyundai car, but there might not be a seller in

Georgia, nor might the government of Georgia permit him to import it, then what? Well, there is a demand for a Hyundai car, but it cannot be met. In other words, it is an unrealizable, in terms of purchasing, demand.

Demand for a good depends on several factors, and it changes as any one or more of these factors change. However, it is relevant to recall here the two important determinants of demand - own price and time.

1.3. Definition of Demand

Demand is usually defined as a schedule which shows various quantities of a product which one or more consumers are willing and able to purchase at each specific price in a set of possible prices during a specific period of time.

1.3.1. Definition of demand by Alfred Marshall

Marshall's (1920) suggestion that the influence of demand on price determination is relatively easy to analyze may well be correct. Yet there were problems with the theory of demand that Marshall (1920) was not able to solve satisfactorily. He seemed to recognize these difficulties and avoided them by assumption. His most important contribution to demand theory was his clear formulation of the concept of price elasticity of demand. Price and quantity demanded are inversely related to each other; demand curves slope down and to the right. The degree of relationship between change in price and change in quantity demanded is disclosed by the coefficient of price elasticity.

Because price and quantity demanded are inversely related, the computed price elasticity of demand coefficient would be negative. By convention, to express the coefficient as a positive number, a negative sign is added to the right side of the equation. The price of a product times the quantity demanded will equal the total expenditure of the buyers or, alternatively, the total revenues of the seller. If price decreases by 1 percent and quantity demanded increases by 1 percent, total expenditure, or revenue, will remain unchanged and the coefficient will have a value of 1. If price decreases and total expenditure or revenue increases, the coefficient will have a value greater than 1 and the commodity is said to be price elastic. If the price decreases by a given percentage and quantity demanded increases by a smaller percentage, total expenditure or revenue will decrease, the coefficient will have a value less than 1, and the commodity is said to be price inelastic. Marshall (1920), with his mathematical ability, was first to express notion of price elasticity precisely; he is therefore considered its discoverer.

According to Marshall (1920), individuals desire commodities because of the utility received through their consumption. The form of the utility function used in his calculations was additive; that is, he derived total utility by adding the utilities received from consuming each good. The utility received from consuming good A depends solely on the quantity of A consumed, not on the quantities of other goods consumed. Thus, substitution and complementary relationships are ignored.

Marshall (1920) assumed that utility was measurable through the price system. If an individual pays $2 for another unit of good A and $1 for another unit of good B, then A must give twice the utility of B. He also argued that intergroup comparisons of utility were possible because in group comparisons, personal peculiarities are washed out.

In Marshall's framework, the most important task of the theory of demand is to explain the shape of the demand curve. If a commodity's marginal utility decreases as more of the commodity is consumed, does it follow that individuals will pay lower prices for larger quantities? Are demand curves, then, negatively sloped? Marshall accepted diminishing marginal utility (see Gossen(1854) and Laurence(1984) for more on Gossen's First Law) and formulated the equilibrium condition that would give maximum utility for an individual consuming many commodities (see Gossen(1854) and Laurence(1984) for more on Gossen's Second Law).

In equilibrium, the consumer will spend in such a way that the last dollar spent for any final good will have the same marginal utility as that spent for any other good. The ratios of these marginal utilities to prices will be equal to, and thus disclose, the marginal utility of money. The marginal utility of money is the marginal utility received from the last dollar of expenditure. If saving is considered as a good, then the marginal utility of money is the utility received from the last dollar of income. The marginal utility of a single good is equal to its price times the marginal utility of money.

Let's move through the derivation of a demand curve in order to see some of the problems encountered and Marshall's (1920) solution to these problems. If we begin with an individual who is maximizing utility and then lower the price of one good, we can derive the relationship between price and quantity demanded. Lowering the price of good A will lead to an increase in quantity demanded only under certain conditions. Lowering the price of good A will have two effects. The substitution effect reflects the fact that good A is now relatively cheaper than its substitutes, and so the individual's consumption of good A will increase. The substitution effect will always lead to greater consumption at lower prices and less consumption at higher prices. The income effect produced by price changes is more complex. Lowering the price of good A increases an individual's real income. With the lower price, the individual can buy the same quantity of good A as before and have income left over that can be spent on good A or on other goods. A normal good is one whose consumption increases with increases in income. If good A is a normal good, its demand curve will slope down and to the right. Lowering its price will increase the quantity demanded through both the substitution effect and the income effect.

If good A is an inferior good, other complications occur. An inferior good is one whose consumption decreases with increases in income. Hamburger might well be an inferior good in a consumer's budget. As income increases, the quantity of hamburger consumed will decrease as better cuts of beef replace hamburger. If good A is an inferior good, then a fall in its price will lead to an increase in its consumption because of the substitution effect, but a decrease in its consumption because of the income effect. If the substitution effect is stronger than the income effect, the demand curve will be negatively sloped; but if the income effect is stronger than the substitution effect, the demand curve will be positively sloped. The possibility of upward-sloping demand curves is extremely disturbing to the theory of demand. The theoretical possibility exists, but no empirical information has yet been produced to indicate the actual occurrence of upward-sloping demand curves.

Marshall (1920) first stated the general law of demand: "The amount demanded increases with a fall in price, and diminishes with a rise in price." He then noted that information gathered by Robert Giffen suggests that the demand curve of poorer individuals for bread may slope up and to the right. In other words, for these individuals, a rise in the price of bread results in a reduction in the consumption of meat and of more expensive foods, and a rise in the consumption of bread. For this reason, inferior goods with a more powerful income effect than substitution effect are referred to as Giffen goods in the theoretical literature. Again, although there is a considerable body of theoretical literature on the so-called Giffen paradox, no acceptable statistical information showing actual upward-sloping demand curves has been documented.

Let's get back to the theoretical problems of deriving demand curves and how Marshall (1920) handled them. Because he worked with an additive utility function, he ignored substitution and complementary relationships in his formal mathematical treatment of deriving demand curves—although, characteristically, he did discuss these issues. He simply assumed that the income effect of small price changes is negligible; that is, that the marginal utility of money remains constant for small changes in the price of any single commodity.

Marshall (1920) had two reasons for dismissing these theoretical difficulties by assuming that the marginal utility of money was constant: first, he did not have the theoretical tools to distinguish clearly between the substitution and income effects; second, he claimed that the income effect of minor changes in the price of a good was so small that no harm was done by ignoring it.

1.3.2. Definition of demand by Léon Walras

Patinkin (2008) describes Walras' Law as the sum of the values of excess demands across all markets must equal zero, whether or not the economy is in a general equilibrium. This implies that if positive excess demand exists in one market, negative excess demand must exist in some other market. Thus, if all markets but one are in equilibrium, then that last market must also be in equilibrium.

This last implication is often applied in formal general equilibrium models. In particular, to characterize general equilibrium in a model with *m* agents and *n* commodities, a modeler may impose market clearing for *n*-1 commodities and "drop the *n*-th market-clearing condition." In this case, the modeler should include the budget constraints of all m agents (with equality). Imposing the budget constraints for all m agents ensures that Walras' Law holds, rendering the n-th market-clearing condition redundant. (Patinkin, 2008)

In the farmer example, suppose that the only commodities in the economy are cherries and apples, and that no other markets exist. If excess demand for cherries is zero, then by Walras' Law, excess demand for apples is also zero. If there is excess demand for cherries, then there will be a surplus (excess supply, or negative excess demand) for apples; and the market value of the excess demand for cherries will equal the market value of the excess supply of apples. (Patinkin, 2008)

Walras' Law is ensured if every agent's budget constraint holds with equality. An agent's budget constraint is an equation stating that the total market value of the agent's planned expenditures, including saving for future consumption, must be less than or equal to the total market value of the agent's expected revenue, including sales of financial assets such as bonds or money. When an agent's budget constraint holds with equality, the agent neither plans to acquire goods for free (e.g., by stealing), nor does the agent plan to give away any goods for free. If every agent's budget constraint holds with equality, then the total market value of all agents' planned outlays for all commodities (including saving, which represents future purchases) must equal the total market value of all agents' planned sales of all commodities and assets. It follows that the market value of total value of excess demand in the economy must be zero, which is an the statement of Walras' Law. Walras' Law implies that if there are n markets and n-1 of these are in equilibrium then the last market must also be in equilibrium, a property which is essential in the proof of the existence of equilibrium. (Patinkin, 2008)

1.3.3. Contributions to demand definition by other researchers

Generally, a concept of utility was more important than that of a preference. Before Fisher (1892) and Pareto (1896), utility was conceived as fundamental that is, it was assumed to be a measurable scale for the degree of satisfaction of the consumer. Fisher and Pareto were the first to state that an arbitrary increasing transformation of the utility function has no effect on demand. Edgeworth (1881) defined utility as a general function of quantities of all commodities, and employed a concept of indifference curves. Böhm and Haller (1987) note that now it is commonly accepted in demand theory that only ordinal utility matters. Further they conclude that a utility function is merely a convenient device to represent a preference relation, and any increasing transformation of the utility function will serve this purpose as well.

The characterization by utility functions imposes some restrictions on preferences. The issue of characterizing of a preference relation by a numerical function was solved by Debreu (1954, 1959, 1964) based on work by Eilenberg (1941), and by Rader (1963), and Bowen (1968).

Employing cardinal utility, Walras (1874) come up with the first "theory of demand": The demand is a function of all prices and endowment, obtained through utility maximization. Slutsky (1915) finally assumed an ordinal utility function with enough restrictions to obtain a maximum under any budget constraints and testable properties of the resulting demand functions.

Antonelli (1886) was the first to propose the opposite way: constructing indifference curves and a utility function from the inverse demand function. The formation of preference relations from demand functions was achieved in two ways: first, Samuelson (1947) and Houthakker (1950) presented the concept of revealed preference into demand theory. Significant progress in relating utility and demand in terms of revealed preference was realized by Uzawa (1960), further improvements being due to Richter (1966). Second, Hurwicz and Uzawa (1971) added to the following integrability problem: construct a twice continuously differentiable utility representation from a continuously differentiable demand function which satisfies certain integrability conditions (including symmetry and negative semidefiniteness of the Slutsky matrix). Kihlstrom, Mas-Colell and Sonnenschein (1976) incorporated the two approaches relating the axioms of revealed preference to the properties of the Slutsky matrix.

1.4. Types of Demand

The product market in every economy is rich in number of goods and services available to customers. It is important for conduction of the demand analysis for decision making to be able to classify the product market. Also, it is inevitable for decision making the understanding of demand at various levels of aggregation. The studies of Hirschey (2006), McGuigan et al (2010), Gylfason (1999), Froeb and McCann (2009), Spulber (2009), Gupta (2001), Varian (2010) made a substantial contribution to the significant classification in these two respects:

- Demand for consumers' goods and producers' goods
- Demand for perishable and durable goods
- Autonomous (direct) and derived (indirect) demand.
- Individual buyer's demand and all buyers' (aggregate / market) demand.
- Firm and industry demand
- Demand by market segments and by total market

1.4.1. Consumers' goods and producers' goods

Goods and services used for final consumption are called consumers' goods. These include those consumed by human-beings (e.g. food items, clothes, kitchen utensils, residential houses, medicines, and services of teachers, doctors, lawyers, and shoe-makers), animals (e.g. dog food and fish food), birds (e.g. grains), etc. In contrast, producers' goods refer to the ones used for production of other goods. Thus, producers' goods consist of plant and machines, factory buildings, services of business employees, raw-materials, etc.

The distinction according to Gupta (2001) and Varian (2010) is somewhat arbitrary. This is because, whether a good is consumers' or producers' depends on its use. For example, if a sofa set is used in the drawing room of a house, it is a consumers' good; while if it is used in the reception room of a business house, it is a producers' good. Nevertheless, the distinction is useful for a proper demand analysis for while the demand for consumers' goods depends on households' income that for producers' goods varies with the production level, among other things. (Varian, 2010)

1.4.2. Perishable and durable goods

Both consumers' and producers' goods are further classified into perishable (non-durable) and durable goods. In layman's language, perishable goods are those which perish or become unusable after sometime, the rest are durable goods. Thus, the former would include items like milk, fish, eggs, and paper cups and plates; and the latter would include furniture, cars, refrigerators, clothes and shoes. According Gylfason (1999), in economics the meaning of these terms is different. Here, those goods which can be consumed only once are refered to be perishable goods. In other words, these goods are themselves consumed. Those goods which offer their services to consume are referred to be durable goods. Thus, referring to Gylfason (1999), perishable goods include all services (e.g. services of teachers and doctors), food items, raw-materials, coal, and electricity, while durable goods include plant and machinery, buildings, furniture, automobiles, refrigerators, and fans.

Gupta (2001) notes that the distinction is significant, durable products pose more complicated problems for demand analysis than do non-durables. Sales of non-durables are made largely to meet current demands which depend on current conditions. In contrast, sales of durables goods go partly to satisfy new demand and partly to replace old items. Further, the latter set of goods are generally more expensive than the former set, and their demand alone is subject to rescheduling, and postponement, depending on current market conditions vis-a-vis expected market conditions in future.

1.4.3. Autonomous and derived demand

The goods whose demand is not tied with the demand for some other goods are said to have autonomous demand, while the rest have derived demand. Thus, the demands for all producers' goods are derived demands; they are needed in order to obtain consumers or producers goods. So is the demand for money which is needed not for its own sake but for its purchasing power, which can buy goods and services. Similarly, demand for car's battery or petrol is a derived demand; it is linked to the demand for a car. There is hardly anything with demand totally independent of any other demand. But the degree of this dependence varies widely from product to product. For example, demand for petrol is totally linked to the demand for petrol driven vehicles, while the demand for sugar is only loosely linked with the demand for milk. Thus, the distinction between autonomous and derived demand is more of a degree than of a kind. Sometimes a distinction is also drawn between direct and indirect demand, and that distinction is close to the difference between autonomous and derived demand, respectively.

Goods that are demanded for their own sake have direct demand while goods that are needed in order to obtain some other goods possess indirect demand. In this sense, all consumers' goods have direct demands while all producers' goods, including money, have indirect demand. (Gupta, 2001;Varian, 2010)

1.4.4. Individuals' demand and market demand

The demand for a good by an individual buyer is called individual's demand while the demand for a good by all buyers in a market is called market demand. For example, if the milk market consisted of, say, only three buyers, then individuals and market demand (monthly) could be as follows:

Table 1.1
Demand schedule for milk

Milk Price GEL/Liter (1)	Milk Demand by (liters)			
	Buyer 1 (2)	Buyer 2 (3)	Buyer 3 (4)	All buyers' market demand (5)
8	5	10	0	15
7	8	12	4	24
6	12	15	7	34
5	20	19	12	51
4	30	25	20	75
3	45	30	30	105

In the Table 1.1, columns (1) and (2) represent buyer l's demand, columns (1) and (3) buyer 2's demand, columns (1) and (4) buyer 3's demand, and column (1) and (5) the market demand for milk. The market demand is obtained as the sum of all buyers demand at respective prices. The graphical derivation of market demand is illustrated, in Figure 1.1.

Parenthetically, it should be noted here, that since tables are discrete and graphs are continuous, we have drawn graphs here only to approximate the table.

In Figure 1.1, the line formed by D_1D_1 represents demand curve of buyer 1, the line D_2D_2 of buyers 2, the line D_3D_3 of buyer 3, and solid line DD that is the sum of all three, is called the market demand curve. The market demand curve can be obtained graphically through a horizontal summation of individual demand curves.

It is obvious that business firms will be interested in the gross market demand curve which is the demand of all relative representatives for their goods and services. Likewise, each consumer taken alone will be concerned only with his own individual's demand curve.

1.4.5. Firm and industry demand

Most goods today are produced by more than one firm, and thus there is a difference between the demand facing an individual firm and that facing an industry (all firms producing a particular good constitute an industry engaged in the production of that good). For example, bottled mineral waters in Georgia are manufactured by IDS Borjomi Georgia (Borjomi, Likani brands), Healthy Water JSC (Nabeghlavi brand), and Sairme Mineral Waters Ltd. Demand for Borjomi mineral water alone is a firm's (company) demand where as demand for all kinds of bottled mineral water is industry's demand. Similarly, demand for Beko refrigerators is a firm's demand, while that for all brands of refrigerators is the industry's demand.

The distinction is very important, because while there are close substitutes for firms' products, no such close substitutes exist for industry's product. (Hirschey, 2006; McGuigan et al, 2010)

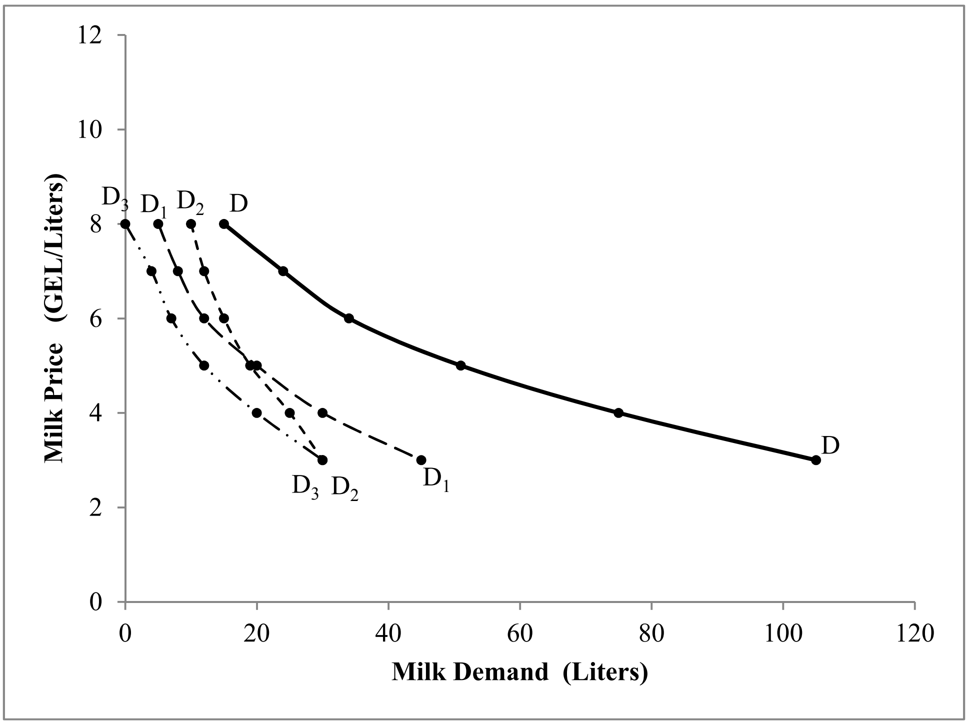

Figure 1.1. Individuals and market demand curves

1.4.6. Demand by market segments and by total market

If the market is large in terms of geographical spread, product uses, distribution channels, customer sizes or product varieties, and if any one or more of these differences are significant in terms of product price, profit margins, competition, seasonal patterns or cyclical sensitivity, then it may be worthwhile to distinguish the market by specific segments for a meaningful analysis. In that case, the total market demand would mean the total demand for the product from all market segments while a particular market segment demand would refer to demand for the product in that specific market segment. For example, one can talk about the domestic demand for Borjomi mineral water versus the total (domestic + foreign) demand for that product, demand for steel for household (kitchen) vis-a-vis its demand for industrial uses, demand for fish by households vis-a-vis that bulk buyers (e.g. hostels, restaurants, hotels, guest houses) and consumption (human) demand for fish versus industrial demand for fish (cattle feed etc.).

The distinction is particularly useful for identifying the problem areas. For example, if a classification is made between domestic and foreign demand, one can analyze the causes of poor foreign market, if that is the case, and find out the ways of augmenting that market. (Hirschey, 2006; McGuigan et al, 2010)

1.5. Determinants of Demand

As Hirschey (2006) and Gylfason (1999) note, the demand analysis is needed basically for three purposes:

- To provide the basis for analyzing market influences on the demand.
- To provide the guidance for manipulating the demand.
- To guide in production or planning through forecasting the demand.

To achieve the above objectives, demand analysis must include the factors which have influence on the demand.

No list of demand determinants could be exhaustive. The major ones would include those listed below in subtopics.

This is not an exhaustive list of the determinants of demand for consumers' products. However, it does include all the significant ones. The remaining factors are basically psychological and climatical, and are therefore difficult to quantify.

It must also be emphasized here that all the determinants are not equally important in the demand for various goods and services. Some variables are significant in demand analysis for some goods, while other variables are important in demands for other goods.

In what follows, the rationale for each of these factors is offered and the influence analyzed. Parenthetically, it may be noted that the analysis will be partial in the sense that, when the analysis is presented for demand with respect to one particular determinant, the other determinants would be assumed to remain constant.

Demand presupposes the existence of the ability to pay for the product. The ability to pay is in turn determined by the income, wealth or/and the credit worthiness of the consumer. Theoretically speaking, these three factors should all be included in the list of demand determinants. However, wealth is a stock variable, and the income would include that from work as well as property, and thus wealth is not taken as a separate argument in the demand function. Similarly, credit worthiness depends heavily on the income and wealth of a person, and therefore it is also excluded from the exclusive list of demand determinants. Thus, income acts as a surrogate, scale or constraints variable to take care of the ability to pay requirement in the demand function (Gupta, 2001).

Economists (see Hirschey (2006); McGuigan et al (2010); Gylfason (1999); Froeb and McCann (2009); Spulber (2009); Gupta (2001); and Varian (2010)) classify goods into normal, superior, and inferior goods. By definition (Varian, 2010), the former are those whose demand varies directly with income, while the latter are those whose demand varies inversely with income. For example, milk, refrigerator, television, education, and the high quality of food grains and clothes are superior goods, while the poor quality of food grains and clothes are inferior goods. The relationship between demand and income is of this type, because as the consumer's income increases, his purchasing power goes up, and therefore he increases his consumption of superior goods, and if he was consuming some inferior items earlier, he would give them up either totally or partially in favour of superior items. Engel was the first person to study this relationship systematically, and the curve reflecting the relationship between demand and income, ceteris paribus, is known as the Engel's curve.

Lewbel (2006) states the curvature of the Engel curve depends on the degree of the superiority or inferiority of the good in question. Thus, the curve could be linear (straight line), convex or even concave.

1.5.1. Own price and demand (The law of demand)

It is well known and accepted that the demand for a good varies inversely with its own price, ceteris paribus. It means, as the price of the Borjomi mineral water increases, other factors hold constant, the demand quantity for Borjomi mineral water decreases, and vice versa. This is known as the law of demand. Worth mentioning here that the law, which describes the inverse relationship between quantities demanded and price, includes a condition, that is, other things hold constant or in Latin "ceteris paribus". The "other things" here refer to all the factors which affect the demand excluding its own price. Some people make fun of economists by saying that while most prices are increasing, no demand for corresponding price is decreasing, thus meaning that the law of demand does not in effect. However, they forget that the "other things" have not remained constant and also affected the quantity demanded.

Several theories have been developed to explain the law of demand (see for more details Böhm & Haller, 1987). The most convincing one goes through the concepts of price effect, income effect and substitution effect of a price change on its own demand. The price effect (PE) refers to the effect of a change in the price of a commodity, ceteris paribus, on the demand for that commodity. This effect is divided into income effect (IE) and substitution effect (SE):

PE = IE + SE (Gupta, 2001).

The income effect refers to the effect of a price change on its demand, ceteris paribus, arising due to the corresponding change in the (real) income of the consumer (Gupta, 2001). For example, suppose the nominal income of the consumer is 500GEL per month, price of milk is 5GEL per liter and currently he buys 60 liters of milk per month. Now, if the price of milk falls to 4GEL, ceteris paribus, he would save 60GEL (60x5 - 60x4), if he continues buying 60 liters of milk. Alternatively, with the earlier milk budget of 300GEL (60x5), he could now purchase 75 liters of milk instead of earlier 60 liters. Thus, a fall in the price of milk is equivalent to an increase in the consumers' income. Depending on whether milk is a superior or inferior good, its demand would increase or decrease. Thus, the income effect could be negative or positive.

The substitution effect stands for the effect of a price change on its demand, ceteris paribus, resulting from a change in the relative prices of the goods in the consumption basket (Gupta, 2001). For example, if the price of the Borjomi mineral water goes up, other things (including the prices of other bottled mineral waters) remaining the same, the Borjomi mineral water would now become relatively more expensive (compared to other bottled mineral waters), than it was before the price change occurred and therefore some people will switch over from the

Borjomi mineral water to other brands of bottled mineral waters, causing a depression for the demand for Borjomi mineral water. Thus, an increase in the price of the Borjomi mineral water, ceteris paribus, would lead to a decrease in the demand for the Borjomi mineral water through the substitution effect. The substitution effect is thus always negative, in the sense that the price change and the change in demand through the substitution effect are in the opposite direction.

A price change, which produces both the income and substitution effects, thus leads to a change in the demand for the good whose price has undergone a change (Gupta, 2001). The total effect is negative (i.e. the law of demand holds good) under two conditions (Varian, 2010):

- when the good in question is a superior good;
- when the good in question is an inferior good but the positive income effect is weaker than the negative substitution effect.

When the product is a superior item, both the income and substitution effects are negative and hence this sum (total effect) is obviously negative. In a situation when a good in question is an inferior item, the two effects work in the opposite direction and their sum could either be negative or positive. But if the positive income effect is outweighed by the negative substitution effect, the price effect continues to be negative, substantiating the law of demand (Gupta, 2001). The law of demand does not hold well for that inferior good (called the Giffen good, in the honor of Giffen, who first explained it) whose positive income effect is stronger than the negative substitution effect. Thus, for a good to be Giffen, it must be an inferior good, it must have a strong income effect (which would be possible if and only if the consumer spends a significant part of his income on this good) and it must have a relatively weak substitution effect (which would happen if and only if there are no close substitutes for that product). The demand for a Giffen good varies directly with its price and thus it provides an exception to the law of demand (Gupta, 2001).

The law of demand presented in the form of a table is called the demand schedule (Table 1.1), and the one in the form of a graph is called the demand curve Figure 1.1. So long as the law of demand holds good, the demand curve is falling, and depending upon the degree of the relationship between price and demand the curve could be linear, convex or concave (Gupta, 2001). However, Varian (2010) notes that the market demand curve does not touch either the price or quantity axis. It does not touch the price axis because if it did, it would mean that there is some price at which demand is zero, which is never the case, for no matter what the price is, there would be at least some buyer in the market, and no seller would set a price at which there is no demand. Similarly, the demand curve would not cut the quantity axis, because if it did, it would mean that there can be a zero price (free good), which is not possible for any economic good as there are no doubt some production costs, and that even at a zero price, the demand is limited. Parenthetically, Varian (2010) notes that since there is a unique quantity demanded at a unique price at a given point of time, all but one price- quantity combination on the demand schedule / curve are hypothetical ones.

Gupta (2001) remarks the exceptions to the law of demand that could arise even for a non-Giffen good under the following situations:

- when the good in question is a luxury item, having some snob value;
- when the good in question goes out of fashion.

Rich people buy costly items such as diamonds in large quantities in spite of their high prices, and sometimes they increase such purchases in the face of rising prices, for the acquisition of such expensive items to distinguish themselves from common people who cannot afford them. Also, some consumers judge quality by high prices, and if so, they might purchase more of a product when its price is high than when its price was low. Similarly, with the emergence of television, the demand for radios has gone down even though their prices have declined. This is because radios are out of fashion these days.

1.5.2. Prices of related goods and demand

Consumers' goods may have either of the two kinds of relationships: substitutes and complements. The two goods are substitutes in a consumption basket, if either of them could meet the consumption need (Froeb and McCann, 2009). The degree of substitution might vary from product to product. The two goods X and Y are said to be complementary goods if consumer needs good X when he has good Y (Froeb and McCann, 2009). For example, tea and sugar, car and petrol, cigarettes and match boxes are pairs of complementary items. One again, the degree of complementarity might vary from one pair of goods to another. For instance, car and petrol are perfect complements while tea and sugar do not have such a strong relationship.

If the two goods A and B are substitutes, an increase in the price of good B, price of good A remaining constant, would induce consumers to substitute good A for good B, because good B has become relatively more expensive now than before, and thereby increases the demand for good A. Thus, an increase in the price of a substitute good would lead to an increase in the demand for the good in question. Quite the opposite would happen when the price of a substitute item falls. In the case of complementary goods, the relationship is the other way round. If the price of a complementary item goes up, the demand for the parent good goes down; for since the former item has become relatively expensive now, the consumer would like to demand less of it, and since he has less of it now, he would need less of the latter good as well. Similarly, when the price of a complementary good goes down, the demand for its parent good goes up (Froeb and McCann, 2009).

Since an item could have more than one substitute goods, and/or more than one complementary product, each of the substitutes' prices and complementaries' prices is a vector of prices rather than just one price (Froeb and McCann, 2009).

1.5.3. Consumers' tastes and preferences, and demand

Consumers' tastes and preferences are an important determinant of the demands for all consumers' good (Spulber, 2009). If a person is a pure vegetarian, either because of religion, tradition, or taste, his demand for meat is zero no matter what his income and the price of meat are. Similarly, if a product goes out of fashion, or taste, its demand goes down. On the other hand, if an item becomes popular (due to improved taste for it), its demand goes up. Television, car and most luxury items fall in the category of popular items.

Producers spend a lot of money on advertising their products primarily because they can influence the tastes and preferences of the consumers in their favour, and thereby achieve an increase in the demand for their own products. It is essentially this activity which enables firms to manage the demand for their products.

1.5.4. Consumers' expectations and demand

Consumers' expectations with regard to their future income and future prices of the good in question in relation to its substitutes and complementary products exert influence on the demand for many goods (Spulber, 2009). For example, graduate students are often observed to spend beyond their means, for their future incomes are high, while the service people in their late fifties try to cut on non-essentials, for their future incomes are low. Thus, an increase in expected future income leads to an increase in the demand for some consumers' goods and vice versa. Similarly, an increase in the expected future price of a product leads to an increase in the demand for that product in the current period (Varian, 2010). To cite an example, in the month of December, there is generally an upward movement in the demand for many luxury and semi-luxury items (e.g refrigerators, air conditioners, television), for there is always a fear that their prices might go up after the new budget is announced on the last day of December. Also, when consumers fear shortage of a commodity, they are often found to buy and stock under panic. Quite the opposite holds when there are rumors of bumper crops.

The expectations' variable plays a more significant role in the case of demand for durable and expensive items than it does in the case of demand for perishable and cheap products (Spulber, 2009). This is because the purchases of durables can be postponed and rescheduled more easily than those of non-durables, and the price changes matter more in the case of expensive goods than non-expensive ones. For this reason, the expectations' variables are often left out from the list of demand determinants for non-durable and cheap goods (Gupta, 2001).

1.5.5. Number of consumers, their distribution and demand

The aggregate demand for a good obviously depends on the number of consumers. Other factors remaining the same, the larger the number of consumers, the greater the demand, and vice versa. Thus, the demand for almost all the products in Georgia, as well as in the world as a whole, is increasing over time, partly because the population is increasing over time.

Since demand for most products varies from consumer to consumer, distribution of consumers among appropriate categories also exerts an influence on the aggregate demand. For example, demand for a car depends on the distribution of households into rich and poor ones. If the proportion of rich households to the total number of households increases, demand for cars would increase and vice versa.

Similarly, demand for cosmetics would increase, if the proportion of women in the total population increases, and demand for baby food would fall, if the proportion of babies in the total population falls, and so on. The other relevant distributions in this respect could be male and female, smokers and non-smokers, vegetarians and non-vegetarians, literates and illiterates, and so on.

Clearly, all these distributions are never simultaneously important in the case of a study of the demand for any one product. Therefore, the researcher has to try to choose only the appropriate ones for a particular demand.

Pointless to say, neither the population nor its distribution is a relevant determinant of the demand for a good by an individual consumer.

1.6. The Empirical Studies of Demand

The empirical studies of demand are of three broad types: consumer survey (interviews), experimental surveys (market experiments), and statistical studies (regression analysis).

1.6.1. Consumer interview method

Under this method, consumption habits of the consumers are gathered through interviewing them. Interviews commonly conducted on the basis of census or sample. Census based interview covers all past and prospective consumers, while sample based interview covers only a subset of customers, called the sample. Depending on the complexity of the problem, the interviews may be planned orally or through pre-designed questionnaire. These interviews, called surveys, aim to obtain the relevant information on a variety of variables, useful for estimating the demand function for the product under study (Gupta, 2001). For example, the survey may include information on the quantities of the concerned good, bought at different periods at various prices of the product, and its related substitute goods, consumer's income statement, expectations, socio-economic profile, and analysis of likely changes, if any, in the consumer's taste and preferences related to the future, face-to-face present, and past. The information obtained in this way on the systematic basis from an pretty large sample of consumers will allow researcher to formulate and quantify the function of the demand for the product under consideration.

Consumer surveys are concerned largely with purchasers intentions and are involved with sales forecasting rather than providing information for price policy making (Hirschey, 2006). Therefore, under this approach one cannot obtain information on the variables that are under the control of management, but rather this approach reveals intentions of the consumers and they may not be actions reflecting actual situation.

1.6.2. Market experiments method

As an alternative to the previous method, estimation of the market demand function can be conducted by the market experiments. It has two versions: actual and simulated (Gylfason, 1999). In conduction of actual experiment, selling points are placed in different localities and then reactions of consumers are observed and recorded. The placement in different localities would include consumers that have different levels of income, caste and religion, sex, age group, tastes and preferences, etc. Moreover, during the experiments, various prices could be tried to provoke reactions of the consumers to price changes. If such an experiment is carried out with enough attention with regard to the sample of locations and probable prices, it will be possible and no difficult to come up with a demand function, indicating demand quantities of the consumers at different levels of incomes, prices, and other relevant variables in the function.

The conduction of market simulation method, also called “consumer clinic” or “laboratory experiment technique”, involves symbolic money that is distributed to a set of consumers and asking them to shop around in a simulated market. The prices of various goods, their quality, packaging, etc. change during the course of experiments to record reactions of the consumers to such changes. The information obtained through this approach may be sufficient to formulate the function of the demand.

Of these two versions, the actual experiment method is more reliable than the simulated experiment one. This is because the consumers have no stake in the latter and may not take the experiment seriously. As a result the data generated will not be reliable, which would render the whole exercise futile. However, actual experiments, though desirable, might be too costly. Thus, the firm which is interested in following either version would have to debate the pros and cons before deciding.

Controlled experiments can estimate the influence of important demand determinants under management control, but care must be taken to reduce the effect of unimportant variables to a minimum, sometimes stimulated exercises are also undertaken (Gylfason, 1999). The most reliable method of estimating a demand function is to combine experiments with statistical studies (Gupta, 2001). For example, an experiment may be carried out by changing the price of ‘x’ and thereby noting the possible changes in demand for ‘x’; these results may be analyzed further through statistical techniques. Of course, such combined methods are applied more likely by a large firm than by a small firm.

1.6.3. Regression (statistical) method

The most commonly used method for demand estimation followed by economists is the regression method. The method involves four steps (Gylfason, 1999; Gupta, 2001):

- Identification of variables which influence the demand for the good whose function is under estimation.
- Collection of historical data on all the relevant variables.
- Choosing an appropriate form for the function.
- Estimation of the function.

Specification of causal variables comes from the underlying economic theory of demand, discussed earlier in the text of this chapter. However, one should note that a model is a simplified version of a true structure, and the model builder faces a lot of constraints such as availability of data, costs of their collection and time constraint within which one would like to complete the task of demand estimation. For these reasons, a model builder might be content with the important causal variables only. Worth to note, that no important determinants must be ignored, for otherwise the model would be a wrong tool for decision making. Similarly, inclusion of irrelevant variables may also jeopardize the accuracy of estimates. Thus, the model builder has to take utmost care in choosing the causal variables for estimating the demand function (Gylfason, 1999). Needless to say, the units of measurements for various variables would have to be carefully specified.

Once the model has been hypothesized, efforts would have to be put in data collection. The data forms the raw-material for estimation, just as wood forms the raw material for wooden furniture. If wood is of poor quality, the furniture would be of poor quality, thus if data are inaccurate, estimated function would have poor reliability. Data, which are historical, are of two types: time series and cross-section (Gylfason, 1999). Time series data are observations on a variable of a given population over time. In contrast, cross-section data refers to observations on a variable at a point (over a period) of time across different populations.

The estimation could be based on either time series or cross-section data or even on both. Depending upon the availability of data and the problem in hand, the researcher would choose the appropriate data series. Yet another important thing at this stage is the sample size for data (Froeb and McCann, 2009). Thus, the greater the sample size, the more reliable are the estimates. Nevertheless, the quality of data is very important, and so is the cost of data collection both in terms of time and money. This is yet another area where the researcher would have to break his head. Notwithstanding this, there is a minimum size for the sample, which has to be greater than the number of demand determinants. In the absence of this minimum, the regression method of estimation would not be available. If the data on an important variable are not available cither at all or for an adequate sample size, the researcher may have to resort to proxy or dummy variables instead (Froeb and McCann, 2009). For instance, no data on consumers’ tastes and preferences are generally available and researchers usually use time variable as a substitution for it. Similarly, if the demand for the product were affected by the government policy with regard to import of such type of products in the country and that this policy underwent a significant change once during the sample period, then the effect of this variable could be incorporated into the model through a dummy variable, which could take a value of one when the policy is of one kind and a value of zero when the policy happened to be of the other kind. So that Gupta (2001) notes that there are ways by which a careful researcher could salvage the model even in the presence of some data difficulties.

The next stage pertains to the choice of a functional form for the function. The function could take any one of the several forms: linear, quadratic, cubic, double- log, semi-log, reciprocal, etc (Froeb and McCann, 2009). Economic theory, which might give a rough idea about some functional forms, might never be able to identify a unique form (Froeb and McCann, 2009). Under such a situation, the researcher has to experiment with all theoretically plausible forms and then to choose the one which is the most ideal on the grounds of both theory and empirical tests. Be enough to say, that the researcher could estimate the function in a few alternative forms, and then with the aid of some statistical tests and a prior knowledge of some of the signs and magnitudes of the coefficients, choose the most appropriate function. The linear and double-log forms are the most popular forms in the literature (for more details see Hirschey (2006); McGuigan et al (2010); Gylfason (1999); Froeb and McCann (2009); Spulber (2009); Gupta (2001); and Varian (2010)).

Given the theoretical model and the data, the next and the last step is economic estimation. The most popular method available for this purpose is the least-squares method of estimation. It is based on an unconstrained optimization technique. Under the method, estimates of parameters are obtained so that the sum of the squares of the errors between the actual values of the dependent variable and its estimated value is minimized with respect to each of the parameters under estimation. For details on this, one should look up some econometrics text (e.g. Johnston, 1984).

It is obvious that in empirical estimation of demand function, there is no escape from statistical techniques. Much of the usefulness of these techniques depends on the nature of demand data which are available. The limitations on measuring demand may arise due to a number of reasons (Spulber, 2009). Firstly, actual demand relation may be too volatile to be significantly explained by analytical functions. Secondly, models (of demand function) mathematically identified a priori may become under-identified when statistical evidence is gathered. Thirdly, there may be too much multi-co linearity inherent in the variables to allow their separate effects to disentangle. Fourthly, the available data may seriously violate one or more of the statistical assumptions about the error term. Spulber (2009) notes that the empirical demand estimates may not always be reliable. This is not to question the usefulness of empirical demand estimates, but to maintain that we must interpret the available estimates with caution and proceed for fresh estimation with care.

1.7. Derivation of Demand Functions and Curves

1.7.1. Marginal utility approach

Marshall built his demand theory based on two assumptions: 1) an individual assigns a different utility function to each good consumed; 2) the marginal utility of money is constant. This makes it easy to build demand functions because the Marshallian utility functions are not "perfect" representations of the individual preferences (Zaratiegui, 2002).

Marshall (1920: 838-9, note II) establishes the following equilibrium condition for the consumption of good "x":

$$du/dx = d\mu/dm \,.\, dp/dx,$$

that is the marginal utility of any good x (du/dx) must be equal to the marginal utility of money (dμ/dm) per measure of the individual maximum willingness to pay for an additional unit of "x" (dp/dx). Marshall calls it "demand price". It is the derivative of the function F(x) which measures the maximum amount of money that the person would be willing to give for every amount of x.

Nobody will pay an amount of money for a unit of x that implies a utility loss higher than that which is gained from that commodity. In other words, the maximum amount that a consumer would be willing to pay for an additional unit of x will be such quantity that the utility lost in the giving of this amount of money (dμ/dm . dp/dx) will be equal to the utility that will be received instead (du/dx).

Marshall supposes that there is diminishing marginal utility in goods consumption, while that of money is a constant, μ. To this assumption he was driven by his determination to derive welfare consequences from marginal utilities as shown by prices. This can be expressed in equation:

$$U'(x) = \mu . p$$

as well as in the form of $U'(x)/\mu = p$, and, simply, as $f(x) = p$.

The function $f(x) = U'(x)/\mu$ indicates the marginal valuation of the successive units of x. Clearly, the integral of this expression is the function of the total valuation of F(x) that measures directly the most the subject is willing to pay for each quantity of x.

If we are to write the function f(x)=p in the inverse form, $x = f^{-1}(p)$, it can be interpreted as a demand function in the usual sense of the word. That is, a function that indicates the quantity that the subject wants to buy at each price. The demand function has a negative slope, in accordance with the assumptions of Marshall, thus $U''(x) < 0$.

The demand function, obtained in this way, does not depend on the income of the person and on the prices of the other goods. Marshall supposes that the availability of the other goods is reflected in the form of U(x). The availability of what Marshall calls "rival commodities" forces the subjective valuation of a single item of good "x" to change; in other words, forces the marginal utilities or "demand prices" to change.

"The richer a man becomes the less is the marginal utility of money to him; every increase in his resources increases the price which he is willing to pay for any given benefit. And in the same way every diminution of his resources increases the marginal utility of money to him, and diminishes the price he is willing to pay for any benefit" (Marshall, 1920: 96).

This is the most contested aspect of the whole Marshallian demand analysis. It is quite reasonable to assign a lower value to μ for those consumers who are the richest (even if we don't take into account the utility's interpersonal comparisons). Nevertheless, it is not reasonable to suppose that μ doesn't change when the size of total expenditure in any good changes.

The traditional utility analysis is making the following assumptions and propositions:

- Utility is cardinally measurable want satisfying capacity. The level of utility from any act of consumption can be quantified by numbers.
- The measuring scale should be held constant in order to measure the utility. For this reason, marginal utility of money is set constant. Thus, level of utility can be expressed in terms of money.
- Utility of a commodity is independent of others. Thus, there is no relationship between commodities in terms of substitutability and complementary.
- The marginal utility of a commodity X decreases as a consumption of X increases; ultimately marginal utility becomes negative. This is the principle of diminishing marginal utility.
- The ratio of the marginal utility to the price per unit of the commodity is called marginal utility of money spent. Marginal utility analysis is based on the assumption that marginal utility of money remains constant. Therefore the

marginal utility and the price per unit of the commodity must change in same direction and proportion.

According to the "law of diminishing marginal utility", at some point the consumption of additional units will result in smaller increases in utility. The marginal utility of successive units of consumption will always fall. As a result, the demand curve for a good or service is downward sloping. Since the marginal utility of each additional item is less, the seller will have to reduce their price to induce you to buy it.

The simple summation of all consumer demand curves for particular good and service will derive the market demand curve.

1.7.2. Indifference curve approach

The theory of indifference curves was developed by Francis Ysidro Edgeworth (see Edgeworth, 1881), who explained the mathematics needed for its drawing; later on, Vilfredo Pareto was the first author to, in fact, draw these curves (see Pareto, 1919). He was the first economist to draw indifference maps as we know them nowadays. The theory can be derived from William Stanley Jevons's ordinal utility theory, which says that individuals can always rank any consumption bunches by order of preference (see Jevons, 1888).

Indifference curves are lines in a coordinate system for which each of its points express a particular combination of a number of goods or bundles of goods that the consumer is indifferent to consume. This is, the consumer will have no preference between two bundles located in the same indifference curve, since they all provide the same degree of utility. The indifference curves, as we move away from the origin of coordinates, imply higher consumption and, therefore, increasing levels of utility.

An indifference map is a combination of indifference curves, which allows understanding how changes in the quantity or the type of goods may change consumption patterns. Points yielding different utility levels are each associated with distinct indifference curves and are like a contour line on a topographical map. Each point on the curve represents the same elevation.

Consumer theory uses indifference curves and budget constraints to generate consumer demand curves. For a single consumer, this is a relatively simple process. Budget constraints give a straight line on the indifference map showing all the possible distributions between the two goods; the point of maximum utility is then the point at which an indifference curve is tangent to the budget line. The process then continues until the market's and household's marginal rates of substitution are equal.

Now, if the price of one good were to change, and the price of all other goods were to remain constant, the gradient of the budget line would also change, leading to a different point of tangency and a different quantity demanded. These price-quantity combinations can then be used to deduce a full demand curve (Lipsey, 1975). A line connecting all points of tangency between the indifference curve and the budget constraint is called the expansion path (Salvatore, 1989).

Indifference curves analysis of demand behavior of consumer is based on the following assumptions:

- Utility is subjective concept and is measured in ordinal numbers. Utility can be ordered according to scale of preference, however it cannot be quantified. It is more realistic for analyzing the consumer behavior pattern to assume that the situation A is chosen, because A offers more utility than B.
- Utility is dependent. The utilities from various commodities are related, so substitutability and complementarily of commodities must be considered. The utility of a commodity is not measured in isolation without reference to other commodities.
- Consumer is limited by the available disposal resources and faced by the question of choice. Consumer is a multi-commodity consumer and takes into consideration the combination of related goods which is interested to purchase.

The simple summation of all consumer demand curves for particular good and service will derive the market demand curve.

1.7.3. Econometric approach

Econometrics is the use of economic theory, statistical analysis and mathematical functions to determine the relationship between a dependent variable and one or more independent variables. Such relationships, based on past data can be used for forecasting. The analysis can be carried with varying degrees of complexity.

The principle advantage of this method is that it is prescriptive as well as descriptive. This technique has got both explanatory and predictive value. The regression method is neither mechanistic like the trend method nor subjective like the opinion poll method. In this method of estimating, not only time-series data but also cross section data can be used. However, it should be noted that data analysis should be based on the logic of economic theory.

The regression analysis is a statistical technique that can be used to estimate many types of economic relationships, not just demand functions. Regression analysis describes the way in which one variable is related to another. Regression analysis derives an equation that can be used to estimate the unknown value of one variable on the basis of the known value of the other variable.

1.7.3.1. Multiple regression methods

The multiple regression includes more than one independent variable. An advantage of multiple regression over a simple regression is that it can predict the dependent variable more accurately if more than one independent variable is used (Freedman, 2005). Also, if the dependent variable is influenced by more than one independent variable, a simple regression of the dependent variable on a single independent variable may result in a biased estimate of the effect of this independent variable on the dependent variable.

The first step in multiple regression analysis is to identify the independent variables and specify the mathematical form of the equation relating the mean value of the dependent variable to the independent variables (Freedman, 2005).

A difficult problem that can occur in multiple regression is multicollinearity, a situation in which two or more of the independent variables are highly correlated (Armstrong, 2012). If multicollinearity exists, it may be impossible to estimate accurately the effect of particular independent variables on the dependent variable. Another frequently encountered problem arises when the error terms in a regression are serially correlated (Armstrong, 2012). The Durbin-Watson test can be carried out to determine whether this problem exists. Plots of the residuals can help to detect cases in which the variation of the error terms is not constant or where the relationship is nonlinear.

The following researchers, Kayser (2000), Mannering and Winston (1985), Arehibald and Gillingham (1980) had applied multiple regression equation to historical statistical data to form estimated market demand curve.

1.7.3.2. Nonparametric methods

Nonparametric regression is a form of regression analysis in which the predictor does not take a predetermined form but is constructed according to information derived from the data. Nonparametric regression requires larger sample sizes than regression based on parametric models because the data must supply the model structure as well as the model estimates. (Bowman and Azzalini, 1997)

Nonparametric regression is motivated by at least the following four objectives (see Hardle, 1990):

- it provides a versatile method for exploring a general relation between variables;
- it give predictions without reference to a fixed parametric model;
- it provides a tool for identifying spurious observations;
- it provides a method for 'fixing' missing values or interpolating between regressor values.

The semi-parametric regression includes regression models that combine parametric and nonparametric models. They are often used in situations where the fully nonparametric model may not perform well or when the researcher wants to use a parametric model but the functional form with respect to a subset of the regressors or the density of the errors is not known. Semi-parametric regression models are a particular type of semi-parametric modeling and, since semi-parametric models contain a parametric component, they rely on parametric assumptions and may be inconsistent, just like a fully parametric model.

There is a growing literature on nonparametric regression and its semi-parametric cousins. Hardle (1990) and Stoker (1991) offer expressive overviews. Newey and Powell (2003) discuss instrumental variable estimation of nonparametric models. Powell et al(1989)'s average derivative estimator assumes the regressors are continuous. Horowitz and Hardle (1996) proposed a semi-parametric model that accommodates some discrete as well as continuous regressors.

The semi-parametric methods mentioned above are lacking in estimating causal effects in a selection setting, because the intercept is suppressed by nonparametric regression. Andrews and Schafgans (1998) suggested a semi-parametric selection model to resolve this deficiency.

The studies of following researchers, Afriat (1967), Diewert (1973, 1985), Lansburg (1981), Varian (1982, 1983, 1985), Chavas and Cox (1997), are contributions to field of usage of nonparametric methods to analyze behavior of consumers with observed data without specifying functional form of the preferences or/and demand function.

1.7.4. Functional forms of demand

The usefulness of both traditional and modern theories of consumer behavior has been recently put under question. The different approaches to utility are remarkable in theory; however they offer little to an applied business analyst to get the true relationships of the complicated real world.

Therefore many have recently followed a practical approach to demand analysis. They put aside the fundamental law of demand, and formulate demand functions directly on the basis of market data. They have focused mostly on demand at the market level, but not on demand at the individual level, and not on a single commodity, but rather on a group of commodities.

The estimation of such demand functions present difficulties. The use of indexes is inevitable when it concerns the aggregation of demand over individuals and over commodities. This approach creates a numerous problems that associated with those indices. Obvious, that there are various other issues related to statistically estimated demand functions. Enhancements in the techniques of econometric science are assumed to solve those issues. The examples of econometrics at act are the constant elasticity demand function or its log-linear transformation. These multivariate functions have been useful for business decision making. A manager, prior to taking or making decision, can accurately anticipate the impact of his strategies by drawing implications from those estimated functions.

1.7.4.1. Distributed lag models of demand

One of the developments is the expression of demand function in dynamic form. Distributed lag model of demand is an example of this.

Distributed lags appear when a price change produces its effect on quantity demanded only after some period of time; so that this effect is not measured at a single point of time, but is distributed over a period of time.

A distributed lag model is a model used to predict current values of dependent variable by employing current values of and past period values of an explanatory variable. It is applied to time series data with an regression equation (see for details Cromwell et al. 1994; Judge et al. 1980).

The general form of the distributed lag model may be expressed in following manner. Suppose time is divided into discrete periods and Q_t is the demand quantity at period t, Q_{t-1} is the quantity demanded during the period t-1, Q_{t-2} is the quantity demanded during the period t-2, P_t

is the price during the period t, P_{t-1} is the price during the period t-1, Y_t is the income during the period t, Y_{t-1} is income during the period t-1, and so on. So, Q_t can be written as

$$Q_t = f[P_t, P_{t-1}, Y_t, Y_{t-1}, ..., Q_{t-1}, Q_{t-2}].$$

It is obvious from the general form of distributed lag model that past behavior is influencing the current demand decision. The past purchases of durable commodities represent a stock of the commodity which affects the current and further purchases. On the other side, past purchases of a consumer non-durable represent a habit which influences the current demand. Additionally, anyone can come up to the same logic with the factors influencing demand like price expectations and past saving.

A widely used model in this context is the model based on partial or stock adjustment principle furnished by Nerlove (1971) as an alternative specification of the distributed lag model. Houthakker and Taylor (1970) have extended this approach to non-durable giving it the name habit reaction principle.

Nerlove's model for estimating demand of durable commodities result in a demand function of the following form:

$$Q_t^* = \theta Y_t, \quad (1)$$

where Q_t^* is the targeted level that is determined by income variable Y_t which is norm (mean) of all consumers at time t. θ is used to measure the existence of a fixed deviation of the income norm (mean) from the target ratio.

The constraints such as the income, inappropriate savings, credit restrictions, etc. will not allow to consumer to get instantly the desired level of durables. According to Nerlove the adjustment of actual to the desired level of demand is spread over the several time periods. Therefore, in any one time period only fraction of the total desired level of demand is obtained, and the acquisition is ongoing process. So, the actual change ($Q_t - Q_{t-1}$) is only a fraction K of the desired change ($Q_t^* - Q_{t-1}$) and may be expressed as

$$Q_t - Q_{t-1} = K(Q_t^* - Q_{t-1}), \quad (2)$$

where K is set to be the coefficient of stock adjustment in each successive period; and satisfies the following condition: $0 < K <= 1$.

Now rearranging both forms (1) and (2), we get:

$$Q_t = \alpha Y_t + \beta Q_{t-1}.$$

Houthakker-Taylor model also rests on the postulate that past behavior has an influence on current decisions. The current decisions are influenced not only by stocks held by the consumer, but also habits formed by the past consumption. They expand the idea of stock adjustments model to non-durable commodities. In this case, demand is a function of income, the price, and consumer's consumption habit. It means that the current demand for non-durable (Q_t) depends on the purchases of commodities in the past, like habit (Q_{t-1}), current price (P_t) and change in price (ΔP_t); current incomes (Y_t) and change in income (ΔY_t). Thus, the demand function takes the following form:

$$Q_t = A + \alpha P_t + b\Delta P_t + cY_t + d\Delta Y_t + eQ_{t-1},$$

where A is the constant and a, b, c, d and e are the parametric coefficients from which the structural parameters may be derived.

1.7.4.2. Linear expenditure system (LES)

Another development has been the linear expenditure system which is derived from the Klein-Rubin utility function and also referred to as the Stone and Geary utility function (see for details Stone (1954); Pollak and Wales (1969)).

In these models yield a system in which expenditures of individual commodities are expressed as linear functions of total expenditure and prices. This system is compatible with three conditions generally imposed on demand functions, which may be stated as: 1) Additivity that is the sum of expenditures is identically equal to total expenditures; 2) Homogeneity that is each commodity has the sum of the total expenditure elasticity and all the price elasticities is equally zero; (3) Symmetry of substitution matrix that is the Slutsky condition.

These models deal with group of commodities instead of individual commodities. The aggregation of those groups yields total consumer expenditure. The demand function is derived while after the utility function is used to formulate the linear expenditure systems. The additive utility function is determined assuming that complementarity between commodities. Purchased commodities acquired are grouped into wide-range types. As a result, each group should include its respective substitutes and complements. So substitution between groups is lined out. However, substitution can happen also within each group.

The logic of the process is as follows, the consumers buy identical minimum amount of commodities from each group called subsistence requirements that are required for survival. The amount of income left after purchasing the subsistence requirements is allocated between the numerous groups on the in relation to prices.

Application of this method splits the income of the consumer into two parts:

- subsistence income that is the income used to purchase survival needs;

- supernumerary income that is the income disposable after survival needs are met.

A classic example of linear expenditure system is the model formulated by Stone (1954) which is also referred as Stone-Geary utility function. In this model the consumer wants to maximize the utility function written as

$u = \ln u = \Sigma_i \alpha_i \ln(q_i - \gamma_i)$,

where q_i is the quantity of good i, $0<\alpha_i<1$, $\Sigma_i\alpha_i=1$, $\gamma_i>0$, and $q_i-\gamma_i>0$.

Maximization operation is subject to the budget constraint $\Sigma_i p_i q_i = Y$ and yields the following demand function expressed by Stone (1954):

$Q_i = \gamma_i + \alpha_i [(Y - \Sigma_j p_j \gamma_j)/p_i]$, (i,j = 1, …, n)

where Q_i is the quantity demanded of good i, γ_i is the subsistence level of good i, α_i is the marginal budget share, Y is the total expenditure of the consumer, p_i is the price index of good i, $p_i\gamma_i$ is the minimum expenditure to get minimal subsistence level and $Y - \Sigma_j p_j \gamma_j$ is the supernumerary expenditure which is allocated between the goods in the fixed proportions α_i.

1.7.4.3. The Rotterdam model

The Rotterdam model was formulated by Barten (1964) and Theil (1965). It has introduced creditable connections with the economic theory of the consumer and its simplicity has played a very important to its popularity. The Rotterdam model has come up as turning point because it offered many features not presented in previous modeling efforts.

The Rotterdam model is expressed by a double-logarithmic system of inconsiderable changes and utilizes an old tradition in applied demand analysis which specifies the demand functions with no reference to any utility function.

The Rotterdam model is derived from differentiating a double logarithmic demand function $log x_i = a_i + \eta_{iy} log y + \sum_{j=1}^{n} \eta_{ij} \, log p_j, \quad i = 1, \dots, n$, where a_i, η_{iy}, and η_{ij} are constant coefficients; η_{iy} is the income elasticity of demand for good i, $\eta_{iy} = d \log x_i = d \log y$; and η_{ij} is the uncompensated cross-price elasticity of good i, $\eta_{ij} = d \log x_i = d \log p_j$, including both the income and substitution effects of the changes in prices, so that

$$\omega_i dlogq_i = \sum \Pi_{ij} dlogp_j + \mu_i \sum \omega_j dlogq_j$$

where $\mu_i = p_i \left(\frac{\delta q_i}{\delta x}\right)$ is total expenditure elasticity and Π_{ij} is the uncompensated cross price elasticity.

1.7.4.4. The indirect translog model

The translog second-order Taylor approximation is applied to approximate an indirect utility function that is used to derive the indirect translog demand model. The indirect utility function

log u = f(log p_1, . . ., log p_n, log x)

is approximated in second order in order to obtain the following utility function:

$$log\, u = a_0 + \sum_i a_i \, log\left(\frac{p_i}{x}\right) + {}^1/_2 \sum_i \sum_j \beta_{ij} \, log\left(\frac{p_i}{x}\right) log\left(\frac{p_j}{x}\right),$$

where α_0, α, and β are parameters. The resulted equation was formulated by Christensen, Jorgenson, and Lau (1975) and it is a second-order Taylor approximation to any arbitrary utility function. In order to obtain the system of demand equations the Roy's identity is applied to previous equation, so that

$$\omega_i = \frac{a_i + \sum_j \beta_{ij} log\left(\frac{p_j}{x}\right)}{\sum_j a_j + \sum_j \sum_i \beta_{ij} log\left(\frac{p_i}{x}\right)} \qquad (i, j) = 1, \ldots, n).$$

The restrictions of the indirect translog model like additivity, homogeneity, and symmetry can be derived. Deaton and Muellbauer (1993) remark that principal limitations of the indirect translog model for estimating a demand system lie on the required amount of structural parameters and the accuracy of the approximation of only in the locality of some point. It will produce unreliable outcomes on approximation over an entire sampling period or over an entire sample.

1.7.4.5. The Almost Ideal Demand System (AIDS) model

Deaton and Muellbauer (1980) proposed a demand system called Almost Ideal Demand System (AIDS). AIDS gives an arbitrary first-order approximation to any demand system; satisfies the axioms of choice exactly; aggregates perfectly over consumers; it has a functional form; simple to estimate, largely avoiding the need for non-linear estimation; and can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters. Many of these desirable properties are incorporated by the models as Rotterdam or translog, however none of them possesses all of them simultaneously as AIDS.

The widespread application of the AIDS model to estimations of many demand system is attributed to the common representation of market demand as the result of decisions by a rational representative consumer.

The PIGLOG class expressed as cost or expenditure function is a base of AIDS. The cost function defines the minimum expenditure to reach a specific utility level at given prices. Deaton and Muellbauer (1980) defined a cost function for utility u and price vectors p as

$$\log c(u, p) = (1-u) \log\{a(p)\} + u \log\{b(p)\},$$

where a(p) and b(p) are functions of prices.

Therefore it happens that consumer's utility is between 0 (subsistence) and 1 (bliss). The positive linearly homogeneous functions a(p) and b(p) are the costs of subsistence and bliss, respectively.

In order to obtain a cost function of a flexible functional form, so we can define a(p) and b(p) by

$$\log a(p) = \alpha_0 + \sum_k \alpha_k \log p_k + ½\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j$$

and

$$\log b(p) = \log a(p) + \beta_0 \Pi_k p_k^{\beta k} \qquad (j, k = 1, \ldots, n),$$

where α_i, β_i, and γ^*_{kj} are parameters. β_k is the weighted parameter of the price of good k. Consequently, the flexible functional form should possess enough parameters so that its derivatives of β_k can be set equal to the derivatives of an arbitrary cost function. The first equation can be rewritten with substitution of the last two equation so that we get the AIDS cost function:

$$\log c(u, p) = \alpha_0 + \sum_k \alpha_k \log p_k + ½\sum_k \sum_j \gamma_{kj} \log p_k \log p_j + u\beta_0 \Pi_k p_k^{\beta k}.$$

The demand functions can be derived directly from the last equation. The fundamental property of cost function is that its price derivatives are quantities demanded. So, $\delta c(u, p)/\delta p_i = q_i$. Multiplication of both sides by $p_i/c(u, p)$ we find w_i the budget share of good i. Logarithmic differentiation of it results in the budget shares as a function of price and utility:

$$w_i = \alpha_1 + \sum_j \gamma_{ij} \log p_j + \beta_i u \, \beta_0 \, \Pi_k P_k^{\beta k}.$$

The indirect utility *u* can be derived by inversion of the equality of the cost function c(u, p) to x which is the total expenditure of utility-maximizing consumer. Substituting the result of the inversion into last equation, we obtain the AIDS:

$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \{x/P\}$,

where α, β, and γ are parameters, and price index P is defined by

$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j$.

1.7.4.6. BLP method

Rasmusen (2007) is describing one of the recent developments in demand estimation techniques named after Berry, Levinsohn & Pakes (1995) as the BLP method. This method explores the structural demand estimation using the random-coefficients logit model. The demand is estimated based on the individual demand coefficients of each customer. Consequently, the individual choice of each representative consumer from different demographics is used to estimate the demand. Rasmusen (2007) concludes that the estimation part of BLP method estimates the importance of the product characteristics, consumer characteristics, and prices using the generalized method of moments. This method is very flexible, requires weaker assumptions than maximum likelihood method. However, requires a large number of observations and much computing power.

BLP method employs following ideas (see Rasmusen, 2007 for details):

- Instrumental variables are used to count for the endogeneity of prices.
- Product characteristics are taken into account instead of the products themselves. So that it measures the probability of buying rather than measuring how much purchased.
- Looking into the consumer and product characteristics interaction in order to get more accurate measures of the arrangement of consumer purchases substituting from one product to another.
- Resulting in structural estimation rather than just looking at the conditional correlations of relevant variables.
- The difficult optimization problem of estimating parameters that are averaged across consumers is dealt by employing a contraction mapping.

- Instead of applying a single algorithm to estimate the parameters of the model, separate algorithms are used to numerically estimate parameters that enter linearly and nonlinearly.
- All other parameters are estimated by use of the generalized method of moments.

Ideas mentioned above are not all developed by the BLP method. Only fifth and sixth ideas are specific to BLP. However, combination of the all ideas is the power of the BLP method.

The BLP method allows estimation of not only simple demand problems but can be used for variety estimation problems in industrial organization. It does not require much theoretical modeling so that allows testing many different kinds of firm and consumer behavior. However, this flexibility is gained by the considerable complexity. The BLP method is structured by two parts: modeling and estimation. The consumer and product characteristics are taken into account to build up a logit model of a maximizing consumer's choice of product. This is the modeling part, which is a structural model. It can be applied to any structural model of maximizing choice, by consumer, government, or firm. The estimation part employs the generalized method of moments to evaluate the importance of the product characteristics, consumer characteristics, and prices. The generalized method of moments attains the flexibility to the BLP method by requiring weaker assumptions than maximum likelihood. However, it should be noted that like the procedure of maximum likelihood the GMM requires large number of observations and much computational power.

1.8. The Objectives of the Research

The reviewed literature presented in this chapter provides different approaches to solve the estimation of the demand in the market. There are two main approaches to this issue: one is the traditional and modern theories of consumer behavior; whereas the second one is a pragmatic approach to demand analysis. The latter, omits the theoretical frames of the fundamental law of demand, formulates demand functions directly on the basis of observed market data.

The traditional and modern concepts of consumer behavior are theoretically impressive and useful for the academic and in-class simulation of the demand-price relationship; however, it has no practical value for managers of the business firms in dealing with complexity of the real world. Mostly, this approach employs the linear relationship of the demand and price in outlining the fundamentals of the theories.

The pragmatic approach is based on functional forms of the demand analysis. They are useful in making managerial business decisions. However, they are difficult to estimate, requiring much computational power of the analyst as well as specific statistical software. Mostly, this approach takes into account nonlinear forms of the demand and price relationships: distributed lag models, indirect translog model, and etc.

The reviewed literature of existing theories and methods do not provide simple and useful tool to obtain optimal parameters for revenue maximization. The true total revenue maximization requires estimating of market demand curve on the entire domain. It is well known that the entire market demand curve almost in all cases is of essentially nonlinear form. The latter implies the fact that it is impossible to estimate optimal parameters of total revenue maximization by means of local linearization. Linearization of the demand curve can be applied locally, but not to the whole domain of market demand curve (globally).

The described situation predetermines the main objective of the present research: elaborating theoretical foundations and such mathematical model of essentially non-linear demand-price relationship which permits easy and reliable estimation of all its parameters and eventually, reliable estimation of optimal revenue and profit.

The current research study is contributing to the field of pragmatic approach by an assumption that market demand has an essentially nonlinear demand-price relationship, which is consisting of a set of linear demand-price components. This assumption enables us to develop a model to recover linear components of the entire nonlinear market demand employing the predetermined input data obtained by pre-investigation of the market. The developed model is applicable to determine the entire market demand from observed linear components, or to determine linear components from the observed entire market demand.

The following objectives are outlining the research to be conducted:

- Representation of observed demand-price relationships as the integrated essentially non-linear object, consisting of a set of linear demand-price components;
- Elaboration of the discrete and continuous theoretical models of integrated non-linear demand-price processes;
- Elaboration of the new statistical method of estimation of parameters for integrated demand discrete model. Testing of the method throughout several numerical examples;

- Comparison of the integrated demand discrete model results to the conventional polynomial regression analysis methodology;
- Creation of the appropriate software tools;
- Based on integrated demand discrete model, elaboration of the new approach to the estimation of optimal microeconomic parameters;
- Elaboration of the new methodology of determining maximums of integrated total revenue curves;
- Elaboration of the new expressions for the estimation of the optimal values of prices, demand quantities and maximums of total revenues.

CHAPTER 2. REGRESSION MODEL OF INTEGRATED DEMAND

2.1. Basic Definitions, Notations, Terminology, and Assumptions

The current study offers absolutely new concepts to demand analysis that require introducing new terminology in order to distinguish concepts of the current study from existing ones. This section of the current chapter provides basic definitions, notations, terminology, and assumptions to guide through the demand analysis concept under consideration.

Observed Demand *D* – measured demand data obtained by consumer surveys, market experiments, or statistical methods.

Smoothed Demand D_s – demand estimated by regression equation (possibly nonlinear), on the base of Observed Demand *D*.

ε_D – error of measurement of the Observed Demand *D*; very important assumptions about ε_D (as well as other observational errors) will be discussed later.

We assume that between these values exists the following relationship:

$$D = D_s + \varepsilon_D, \tag{2.1}$$

that is that the Observed Demand can be represented by the Smoothed Demand and additive observation errors. This is an assumption typical for such kinds of problems (mostly in regression models). It is important to note that the Observed Demand *D* depends on the Price *P* and this relationship is not linear. The relationship has the unknown form, meaning that the $D= D(P)$ is not specified. Smoothed Demand is an approximation of this relationship, and the *D* and *P* in their nature are different: *D* is the random value, whereas *P* is not. It means that *D* is measured by the error and *P* is not. The notion ε_D in this case represents that error.

Now let's mention assumptions concerning ε_D. We assume that for each measurement of the Observed Demand, the ε_D is a random value distributed as $N(0,\sigma^2)$; it is independent from the variable *D*, which means that measurements of demand both at low and high levels of values have the same accuracy.

Elementary Observed Demand D_i (i=1,2,….,r) – observed demands obtained by consumer surveys, market experiments, or statistical methods, the sum of which is equal (but not necessary!) to the Observed Demand *D*.

Smoothed Elementary Demands D_{si} (i=1,2,…,r) – are demands represented by the regression equations (we assume they are linear for the sake of current study, unlike nonlinear equations of the Smoothed Demand D_s) based on Observed Elementary Demands D_i. The equations have the following form:

$$D_{si} = d_i - a_i P, \ (a_i \geq 0; d_i > 0) \ . \qquad (2.2)$$

It should be noted that estimations of regression equations of elementary demands are done independent of each other.

It is assumed that each of the Elementary Observed Demands has the additive model:

$$D_i = D_{si} + \varepsilon_{Di}, \qquad (2.3)$$

where:

ε_{Di} (i=1,2,…,r) – error in measurement of Elementary Observed Demand D_i.

Assumptions for error are the same as previous one: ε_{Di} is a random value distributed as $N(0,\sigma^2)$; it is independent from the variable D_i, which means that measurements of demand both at low and high levels of values have the same accuracy.

Integrated Demand $D_{A\Sigma}$ – the vertical[1] sum of the smoothed elementary demands. This is smoothed (by the vertical summation) Observed Demand D, which causes the following error:

$$\varepsilon_{\text{DA}} = \sum_{i=i_0}^{r} \varepsilon_{Di} \ , \qquad (2.4)$$

where

ε_{DA} – error of the integration model used to measure Observed Demand. It can be equal to ε_D.

It is assumed that Observed Demand D is

$$D = D_{A\Sigma} + \varepsilon_{DA}. \qquad (2.5)$$

We consider two different representations of Observed Demand D: by regression model (2.1) and by Integrated Demand $D_{A\Sigma}$ (2.5).

The Fig.2.1 represents the above mentioned conceptions.

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[1] Traditionally in economics literature the coordinate system Price-to-Demand is used, whereas hereafter we shall use Demand-to-Price system.

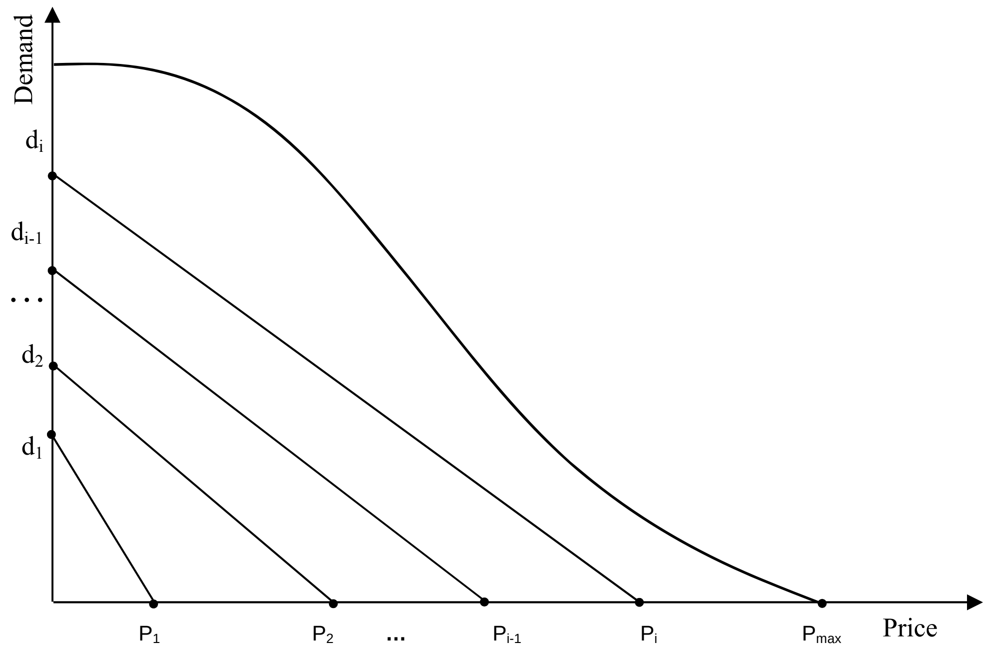

Figure 2.1. Graphical representation of integrated demand $D_{A\Sigma}$

The curve is the smoothed nonlinear demand D_s, straight lines are the smoothed elementary linear demands D_{si}.

Here P_i are prices for the integrated demand model, where elementary demand values are equal to zero, i.e. they are the backup prices of elementary demands. They may be equally spaced, or they may not be. Further, we introduce unitless prices:

$$x = \frac{P}{P_{\max}} \left(x_i = \frac{P_i}{P_{\max}} \right). \tag{2.6}$$

$P_{\max}$ – where the value of price of nonlinear demand curve is equal to zero.

It is obvious from this expression that $0 \le x \le 1$ and $x_{max} = 1$. Besides we introduce one more variable $z = x$ (also unitless price), which will be used for technical purposes in integration.

2.2. General Relationships

Let us define the integrated demand model. The integrated demand model, being the summation of elementary demands, can be represented as follows:

$$\mathrm{D}_{A\Sigma}(P) = \sum_{i=i_0}^{r} (d_i - a_i P), \tag{2.7}$$

where:

i_0 – value of index that is equal to the index of P_i which satisfies $\min_{P_i}((P_i - P) \geq 0)$.

The slope a_i of function (2.7) is defined by the ratio of demand d_i to price P_i, then the equation (2.7) can be rewritten as:

$$\mathrm{D}_{\mathrm{A}\Sigma}(P) = \sum_{i=i_0}^{r} d_i (1 - \frac{P}{P_i}), \qquad (2.8)$$

or, using the unitless price it can be rewritten as:

$$\mathrm{D}_{\mathrm{A}\Sigma}(z) = \sum_{i=i_0}^{r} d_i (1 - \frac{z}{x_i}). \qquad (2.9)$$

All three equations (2.7, 2.8, 2.9) are representations of the integrated demand model. They are equivalent to each other, differing only in the written form, and the last one is more preferable.

Let us discuss the equation (2.9). Firstly, lower limit of summation varies depending on z (x and z represent relative price, in mathematical terms they are different: x_i is the backup price, z is the current price from interval (0, z_{max}=1)). Secondly, backup prices are fixed known values. Thirdly, as unknowns to be calculated one can consider either $D_{A\Sigma}$ or d_i*'s*. Depending on the latter statement the two mutually inverse related problems can arise that will be noted as Direct and Inverse problems.

Statement of Direct problem. Given smoothed elementary demands D_{si} or the set of parameters of d_i, it is required to define smoothed demand D_s, which assumed to be equal to integrated values of $D_{A\Sigma}$. It is clear that the problem can be solved by simply calculating the value of $D_{A\Sigma}(z_i)$ for given relative price z_i.

Statement of Inverse problem. Given observed elementary demands D_i for some values of z_j (i=1,….,n) and backup prices x_i (i=1,….,r) on the interval (0,x_{max}=1). It is required to define values of d_i for the smoothed elementary demands D_{si}. Smoothed elementary demands D_{si} are fully defined through d_i with already given x_i values.

Assume that there are r intervals Δ_1=(0, x_1), Δ_i=(x_{i-1}, x_i),…,Δ_r=(x_{r-1}, x_r=1) (note that x_r=x_{max}=1) set by r values of the backup prices, and let us suppose that values z_j (j=1,…,n) are distributed in such a way that at least one of them falls into one of the intervals Δ_i. Let us define the quantity of z_j points falling into i^{th} interval, through k_i. It is obvious that values of k_i should satisfy the equality $\sum_{i=1}^{r} k_i = n$. Thus, there are r groups of z_j points randomly distributed in intervals Δ_i with one condition: "at least one of them falls into one of the intervals Δ_i". Note that some of them may coincide with the backup points. There are r unknowns d_i*'s*, to find which we need $n \geq r$ values of z_j and $D(z_j)$. This results in a system of linear equations with matrix A_{ij} :

$$D(z_j) = A_{ji} d_i . \text{ (i=1,…,r; j=1,…,n).} \tag{2.10}$$

The matrix A_{ji} is rectangular for $n > r$ and square for $n = r$.

It is not difficult to conclude that the matrix A of the system (2.10) for $n > r$ has the following form:

$$A = \begin{vmatrix} (1-\frac{z_1}{x_1}) & \cdots & (1-\frac{z_1}{x_i}) & \cdots & (1-\frac{z_1}{x_r}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (1-\frac{z_{k_1}}{x_1}) & \cdots & (1-\frac{z_{k_1}}{x_i}) & \cdots & (1-\frac{z_{k_1}}{x_r}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & (1-\frac{z_{k_{i-1}+1}}{x_i}) & \cdots & (1-\frac{z_{k_{i-1}+1}}{x_r}) \\ 0 & \cdots & (1-\frac{z_{k_i}}{x_i}) & \cdots & (1-\frac{z_{k_i}}{x_i}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & (1-\frac{z_{k_{r-1}+1}}{x_r}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & (1-\frac{z_{k_r}}{x_r}) \end{vmatrix} \tag{2.11}$$

and for $n = r$:

$$A = \begin{vmatrix} (1-\frac{z_1}{x_1}) & \cdots & (1-\frac{z_1}{x_{r-1}}) & (1-\frac{z_1}{x_r}) \\ 0 & (1-\frac{z_2}{x_2}) & \cdots & (1-\frac{z_2}{x_r}) \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & (1-\frac{z_r}{x_r}) \end{vmatrix} \tag{2.12}$$

The solution in the second case does not present difficulties and since the matrix A in this case is triangular, it can even be written out analytically without employing numerical calculations.

In the first case we have an overdetermined system (the number of equations exceeds the number of unknowns), and the ordinary least squares method should be used to determine the d_i's, while in the second case we have an $r \times r$ system of linear equations.

It should be noted that we are modeling the dependence of Demand on Price, so that we are dealing with the function of one variable $D=f(P)$, but the system (2.10) represents this dependence as a function of r variables: d_i (i=1,…,r). Besides in the system (2.10) instead of z_j their functions - $(1-\frac{z_j}{x_i})$ are used, which create for j=(1,…,n), referring to the language of regression analysis, a matrix $n \times r$ of observations A represented in (2.11) and (2.12). The estimated parameters are the values of d_i (i = 1, ..., r) that are the intercepts of elementary demands. Thus, the problem of estimating the integrated demand is represented as a linear regression problem for r variables.

2.3. Continuous Integrated Demand

Let us consider next case. Suppose, that quantity of backup points r tends to infinity, and the maximum length of the intervals $\Delta_i=(x_{i-1}, x_i)$ tends to zero. This is a typical construction in mathematical analysis (calculus): a general notion of a definite integral is introduced in a similar way. By employing this technique we get the following statement:

$$D_{SA}(z) = \int_z^1 d(x)(1-\frac{z}{x})dx \tag{2.13}$$

where, the summation sign is converted to the integral, and a vector d_i to a function $d(x)$ [2]. Let us restate the two problems put forward earlier in terms of continuity.

Direct Problem. We are given a function $d(x)$. We are required to define the function of Smoothed Demand $D_s(z)$. The task is simply to calculate the value of the integral $\int_z^1 d(x)(1-\frac{z}{x})dx$ for the current relative price z.

Inverse Problem. Given a function of Smoothed Demand $D_s(z)$, we are required to define the function $d(x)$.

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[2] A function (at least, a sufficiently smooth one) can be viewed as (for the elements of the vector that tend to infinity) a continual generalization of a vector (it is a discrete object). Knowing this, it is easy to understand that an integral is a continual generalization of a discrete sum.

The equation (2.10) can be viewed as some kind of integral transform. In this case, the Direct Problem is interpreted as a direct integral transform and the Inverse Problem is the inverse integral transform. The kernel of this transform is a function $(1-\frac{z}{x})$ [3]. This is an important notification, because despite the tautology of the terminology "direct" and "inverse", we have formulated a problem of economics in in terms of fundamental terminology of mathematical analysis. It should be noted that the second (inverse) problem is, in fact, an integral equation with the kernel $(1-\frac{z}{x})$, and that the solution of the second problem for the discrete case with the matrix (2.11), employing the method of least squares, can be viewed as the quantitative solution of the integral equation (2.13) (assuming that the quantity of backup points r is large enough). The analytical solution of the equation (2.13) has not yet been developed, if any, and is left for the moment for further studies. One can pose a very interesting question about a geometrical interpretation of the transform (2.13), since there is an obvious geometrical relationship between Demand and Price, but let us leave it for further studies.

2.4. Relationship of Parameters of Regression and Continuous-Integrated Demand Model (Solution of the Direct and Inverse Continuous Problems).

To discover a relationship between the regression parameters and Continual-Integrated Demand Model we will do the following. Suppose, that $D_s(z)$ (to remind, we assume that this is a sufficiently smooth function) can be defined in the form of a polynomial of order m.

$$D_{SP}(z) = \sum_{i=0}^{m} a_i z^i \qquad (2.14)$$

The order m, and the coefficients a_i can be obtained through regression analysis.

The coefficients a_i should comply with the equation:

$$\sum_{i=0}^{m} a_i = 0 \qquad (2.15)$$

This is a natural requirement: the demand should be equal to zero at the point z=1.

Similarly we can present the integrand $d(x)$

[3] Due to x≤z, then there is nothing special about it.

$$d(x) = \sum_{i=1}^{m-1} b_i x^{i-1}. \tag{2.16}$$

It should be noted however that, there is no element *x* with a power zero in (2.16), because all free elements (intercepts) should depend on their backup points. Secondly the order of the polynomial *d(x)* is for one unit less than in (2.14). This is explained by the requirement that demands smoothed by polynomial regression (2.14) and by the continual integrated demand (2.13) should be equal. This requirement is the result of continuity: both models, polynomial regression and continuous integrated demand, are continuous functions and results of smoothing empirical data should be the same. It is obvious, that results obtained by discrete integrated demand (2.9) should be different from results of both continual models. It is worth mentioning, looking ahead, that the last statement does not necessarily mean that (2.9) should get some additional error of approximation: if the observed demand is in fact the result of some elementary demands, then the model (2.9) will generate minimal error. This remark we will develop later to check the hypothesis of the structure of the observed demand.

Let us now substitute (2.16) into (2.13) and integrate:

$$\int_z^1 \left(\sum_{i=1}^{m-1} [b_i x^i]\right)\left(1-\frac{z}{x}\right) dx = \underbrace{\int_z^1 \sum_{i=1}^{m-1} b_i x^i dx}_{\textbf{part a}} - z \underbrace{\int_z^1 \sum_{i=1}^{m-1} b_i x^{i-1}}_{\textbf{part b}} \tag{2.17}$$

Part a:

$$\int_z^1 \sum_{i=1}^{m-1} b_i x^i dx = \sum_{i=1}^{m-1} \frac{b_i}{i+1} x^{i+1} \Bigg|_z^1 = \sum_{i=1}^{m-1} \frac{b_i}{i+1} - \sum_{i=1}^{m-1} \frac{b_i}{i+1} z^{i+1}$$

Part b:

$$z\int_z^1 \sum_{i=1}^{m-1} b_i x^{i-1} = z\sum_{i=1}^{m-1} \frac{b_i}{i} x^i \Bigg|_z^1 = \sum_{i=1}^{m-1} \frac{b_i}{i} z - \sum_{i=1}^{m-1} \frac{b_i}{i} z^{i+1}$$

Now let us equate the difference of the right sides of the last two equations to (2.14):

$$\sum_{i=0}^{m} a_i z^i = \sum_{i=1}^{m-1} \frac{b_i}{i(i+1)} z^{i+1} - \left(\sum_{i=1}^{m-1} \frac{b_i}{i}\right) z + \left(\sum_{i=1}^{m-1} \frac{b_i}{i+1}\right) \tag{2.18}$$

It follows that:

$$a_0 = \left(\sum_{i=1}^{m-1} \frac{b_i}{i+1}\right), \tag{2.19}$$

$$a_1 = \left(\sum_{i=1}^{m-1} \frac{b_i}{i}\right), \tag{2.20}$$

and

$$a_{i+1} = \frac{b_i}{i(i+1)}, (i = 1,2, \dots, m-1). \tag{2.21}$$

It is easy to check that a_i defined in this manner will satisfy the requirements of (2.15). Note that by (2.21) we get

$$\sum_{i=2}^{m} a_i = \sum_{i=1}^{m-1} a_{i+1} = \sum_{i=1}^{m-1} \frac{b_i}{i(i+1)}. \tag{2.22}$$

By (2.19) and (2.20) we get

$$a_0 + a_1 = \sum_{i=1}^{m-1} \frac{b_i}{i+1} - \sum_{i=1}^{m-1} \frac{b_i}{i} = -\sum_{i=1}^{m-1} \frac{b_i}{i(i+1)}. \tag{2.23}$$

Summing up (2.22) and (2.23) we get (2.15).

It is obvious that equations (2.19), (2.20) and (2.21) deliver the solutions of Direct and Inverse continual problems, while the following restriction is in force: both functions $D_s(z)$ and $d(x)$ can be presented in a form of polynomial, which can almost always be done in practical cases.

2.5. Errors and Check of the Hypothesis

2.5.1. Errors

Above we have introduced assumptions that observed (measured) demand consists of smoothed demand and additive error, that is $D = D_s + \varepsilon_D$. In previous section we have reviewed two types of equivalent smoothing: by polynomial regression and by continuous integrated demand models. They are equivalent as long as they have equal errors, that is:

$$\mathrm{D} - \mathrm{D}_{SP}(z) = \mathrm{D} - \mathrm{D}_{SA}(z) = \varepsilon_{\mathrm{D}}. \tag{2.24}$$

If smoothing is used for discrete integrated demand (2.9), the error ε_{DA} is different:

$$\mathrm{D} - \mathrm{D}_{A\Sigma}(z) = \varepsilon_{\mathrm{DA}}. \tag{2.25}$$

For further use, we will introduce sum of squares error for the errors listed in (2.24) and (2.25):

1. For ε_D

- $$SSD = \sum_{i=1}^{n}\left[D_i - \int_{z_i}^{1} d(x)\left(1-\frac{z_i}{x}\right)\right]^2 = \sum_{i=1}^{n}\left[\left(D_i - \sum_{j=1}^{m} a_j z^j\right)\right]^2, \tag{2.26}$$

   where the number of degrees of freedom SSD is equal to n-m+1 (number of parameters, considered for evaluation, equal to m: m+1-parameters (this coefficients of polynomial) minus condition of (2.15)), and that is causing standard deviation

- $$MSSD = \sqrt{\frac{SSD}{n-m+1}}. \tag{2.27}$$

2. For $\varepsilon_{\mathrm{DA}} = \sum_{i=i_0}^{r} \varepsilon_{Di}$

- $$SS\Sigma = \sum_{i=1}^{n}\left[D_i - \sum_{j=i_0}^{r} d_j\left(1-\frac{z_i}{x_j}\right)\right]^2. \tag{2.28}$$

The number of degrees of freedom of this summation of squares will be equal to n-r, since the solution requires pre-evaluation of r parameters of d_i. That is why the standard deviation will be equal to

- $$MSS\Sigma = \sqrt{\frac{SS\Sigma}{n-r}}. \qquad (2.29)$$

3. The errors associated with the individual elementary demands ε_D are evaluated by standard methods used for one-dimensional regression analysis, and thus I shall bypass it.

2.5.2. Check of the hypothesis

Direct and inverse problems, besides of the sole calculation aspects that are laid out above, require statistical analysis of the obtained calculated results.

Recall Direct Problem. Given Smoothed Elementary Demands D_{si}, that is the set of d_i parameters, we are required to define Smoothed Demand D_s, which is assumed to be equal to integrated values of $D_{A\Sigma}$, and the solution of the problem is simply the calculation of $D_{A\Sigma}(z_i)$ for the given relative price z_i using formula (2.9).

In this case, we have no hypothesis, so there is actually nothing to be checked. However, before applying formula, we should be sure that each component of the discrete integrated demand is adequate to linear regression of $D_{si} = d_i - a_i P, (a_i \geq 0; d_i > 0)$. This is done by using standard methods of one-dimensional regression analysis that is why I will bypass this part.

What is the meaning of the first problem from economics view point? There is nothing too special about it: it can be used for pure technical application with aim of calculating optimal parameters of market demand such as price, demand, break-even point, etc. However it has also important outcomes, we will present some insights further in this work.

More is contributed by the inverse discrete problem.

Given Demand D_i for some z_i (i=1,…,n) and backup points x_i (i=1,…,r) on the interval ($0, x_{max}=1$), we are required to define values of d_i, for Smoothed Elementary Demands D_{si} (they are completely defined by d_i because values of x_i are known).

Restating the inverse problem in other words: there is matrix of observations of demand and prices (D_i and z_i (i=1,…,n)) and there is a hypothesis[4], that this matrix is not the result of one nonlinear function, but the result of the discrete integrated demand (2.9) with "suspicious" prices x_i(i=1,…,r) [5]. Can this hypothesis be accepted? If the answer is positive, then in the further steps the above mentioned and presented below calculations of optimal parameters can be applied. Besides, this question seems to be interesting in itself, for example from a sociological view point.

Here are the steps of hypothesis check routine.

1. The matrix of data (D_i and z_i (i=1,…,n)) is smoothed by using polynomial regression (with corresponding rank of the polynomial (2.14) - $D_{SP}(z)= \sum_{i=0}^{m} a_i z^i$). This means, that we again employ a standard method, that of polynomial regression. As a result we obtain the values of a_i coefficient as well as the sum of squares required to check this regression for adequacy, including SSD = $\sum_{i=1}^{n}[(D_i - \sum_{j=1}^{m} a_j z^j)]^2$ (2.26).
2. Next, the same matrix of data is smoothed by the discrete integrated demand $D_{A\Sigma}(z) = \sum_{i=i_0}^{r} d_i(1-\frac{z}{x_i})$, employing developed algorithm (refer to appendix for MATLAB code) and measuring all necessary parameters, including SSΣ $=\sum_{i=1}^{n}[D_i - \sum_{j=i_0}^{r} d_i(1-\frac{z_i}{x_j})]^2$ (2.28).
3. Next, we use F-test, that is we calculate the expression

$$F_{n-r,n-m}^{cal} = \frac{\frac{SS\Sigma}{n-r}}{\frac{SSD}{n-m}}, \tag{2.30}$$

and compare it with an appropriate tabulated value of $F_{n-r,n-m}^{tab}$. If $F_{n-r,n-m}^{cal} \geq F_{n-r,n-m}^{tab}$, then the hypothesis, that the observed demand is the result of integrated demand, is rejected, otherwise it is accepted.

[4] Assume that during the marketing research has been noticed existence of some price intervals (0, x_i) (i=1,…,r).

[5] Assume that during the marketing research has been noticed existence of some price intervals (0, x_i) (i=1,…,r).

The logic is the following: to accept the hypothesis of discrete integrated demand the error of smoothing by this integrated demand should be at least equal to, or in best case less than, the error of polynomial smoothing. The statistical significance of the last one is supported by the presented *F*-test. However the application of the *F*-test should be justified. The justification is based on the assumptions on the errors mentioned in first paragraph. That is: for each measurement of observed demand, ε_D and ε_{DA} are the random values, distributed as $N(0,\sigma^2)$ and independent from the value of *D*, as well as from each other. In case of compliance with these two assumptions the *F*-test can be applied for comparing disperse of two random variables with such characteristics. It would be incorrect to apply the *F*-test in existence of mutual dependence between ε_D and ε_{DA}. How is the independence of ε_D and ε_{DA} justified? The answer is in the process of evaluation of the parameters of both polynomial and discrete integrated demand methods. The parameters of both methods are calculated employing the same data by different approach and independently of each other. Thus, the deviations from observed values must have the same probabilistic structure (distributed as $N(0,\sigma^2)$) while being independent from each other.

2.6. Optimal Parameters of Integrated Demand Models

2.6.1. Revenue and profit maximization

The generation of a healthy profit is the main motivation for being in business. Accomplishing that objective will depend on many factors that vary from one business to another. Those factors mainly are the size of the company, the industry to operate in, the profit margin and the volume of sales. The desire to get the most from available resources as possible is the common feature that share business owners and managers, regardless of the field of industry. Profit maximization and revenue maximization are two competing strategies that may be applied to improve the performance of the business.

The business classes start with the concept that any business structure is a profit maximizing organization aimed to gain as much net profit or so called profit as possible with the given resources and share of the market. It sounds quite logical goal for any business firm. However, there may be situations when focusing merely on maximizing profit may lead to forgo opportunities that do not pay off immediately but do offer long-term benefits.

On the other hand, the aim of focusing on the revenue maximization is to control the market share instead of the current profit margin of the sales. This strategy is employed by the new business firms entering the market and the established business firms who are marketing a new product. The revenue maximization is principally appropriate for large competitive markets.

The preference of managers and business owners are to revenue maximization over the profit maximization. Anderson (2011) points out that profit maximization is unrealizable task in the real world due to lack of the information by the managers caused by the uncertainty and the time. More information about the revenue maximization versus profit maximization can be found in the works of Murray N. Rothbard (1962) and William J. Baumol (1967), both of whom hold that firms are revenue maximizers.

It is completely agreed, that profit is the final goal, to be achieved by a business or managers, as it can satisfy all of the stakeholders. But the limitation is that profit is not a physical activity which can be conducted. Profit is a result of an activity like selling and producing. In real world, a manager can focus on revenue maximization rather than profit maximization. There are several reasons that lead to it.

First of all, as mentioned earlier, profit maximization is not a physical activity which can be performed. Other activities affecting profit, which can be practically performed, are sales and production of the goods and services. Production is under managerial control and can be influenced easily. Managerial efforts are required in generating revenues, and therefore revenue maximization can be the strategy to run the business.

Secondly, we know that profit is a function of revenues and costs. Factually, the revenue generation and cost incurrence are not simultaneous activities. Costs are incurred first to perform the production activity, and then sales of the product are initiated. Also, prices are not known when the product is being produced. After the cost incurrence, which may be considered sunk cost, the only objective left is to maximize the revenues as the costs are no longer controlled by the managers.

To add to it, the profit maximization objective is vague. There can be many interpretations of the term. Does it actually mean maximizing profits or minimizing costs? If maximizing profits, in relation to what?

Considering the limitations of profit maximization as an objective, we can safely conclude that revenue or sales maximization is a better and obvious choice for the managers.

In next section we discuss revenue maximization in regard to the model under the consideration. Firstly, an overview of the basic linear demand curve and the corresponding total revenue curve will be presented. Secondly, the nonlinear demand curve cases will be mentioned. Finally, the findings of the integrated market demand model will be discussed.

2.6.2. Optimal values for linear demand curve

The calculation of the price elasticities of demand has a practical application to the real world. The price, quantity of demand and known demand function for the particular commodity is used to define the price elasticity of demand. It should be noted, that there is a relationship between the price charged for the commodity and the total revenue gained from that commodity. Therefore, logically, there must be a close relationship between the price elasticity of demand and total revenue from the sales of the commodity.

The elasticity of the demand for a commodity can be measured by considering the effects of a price change on the total income of the consumer, in another saying, by considering the effect of the price change on the total revenue obtained from sale of the commodity. By the definition of Webster (2003), the price elasticity is a percentage change in the price of a good that results in some percentage change in the quantity purchased (sold) of that good.

Suppose that there is a decline in the selling price of 10% for a specific commodity offered for sale by a firm. This will result in a 10% increase in income of the consumer, and a 10% decline in total revenues of the firm, holding constant quantity demanded. However, the law of demand gets in action and the amount of commodity demanded will not stay constant but in fact, will result in an increase in consumption. Logically, it will result in an increase of total expenditures (revenues) when the percentage increase of demand is greater than the percentage decline in price. In case of the percentage increase in demand is less than the percentage decline in price, the decline in total expenditures (revenues) are expected. Consequently, when the percentage changes in demand and price are equal to each other, the total expenditures (revenues) are expected to stay the same.

The demand (*D*) function for the linear demand curve is represented by following equation:

$$D = b + kP, \tag{2.31}$$

where ($k < 0$) and ($b > 0$).

The Figure 2.2 illustrates the relationship between total revenues and the price elasticity of demand. The quantity of commodity demanded and total revenue increases when the selling price of the commodity is lowered in the elastic region ($|\varepsilon_p|>1$) of the demand curve. If the selling price is to be lowered in the inelastic region ($|\varepsilon_p|<1$) of the demand curve, the quantity demanded increases further, but total revenue experiences the fall. Likewise, when the selling price of the product is increased in the inelastic region ($|\varepsilon_p|<1$) of the demand curve, the quantity demanded falls and total revenue increases. As the selling price is increased in the elastic region ($|\varepsilon_p|>1$) of the demand curve, quantity demanded and total revenue starts falling.

The total revenue (TR) function for the linear demand curve is defined as follows:

$$TR = bP + kP^2 \tag{2.32}$$

Demand curve can be represented as an inverse function of the demand (*D*) function by the following equation:

$$P = b_1 + k_1 D, \tag{2.33}$$

where ($k_1 < 0$) and ($b_1 > 0$), and $b_1 = - b / k$, and $k_1 = 1 / k$.

Both equations (2.31) and (2.33) are representations of linear demand curve.

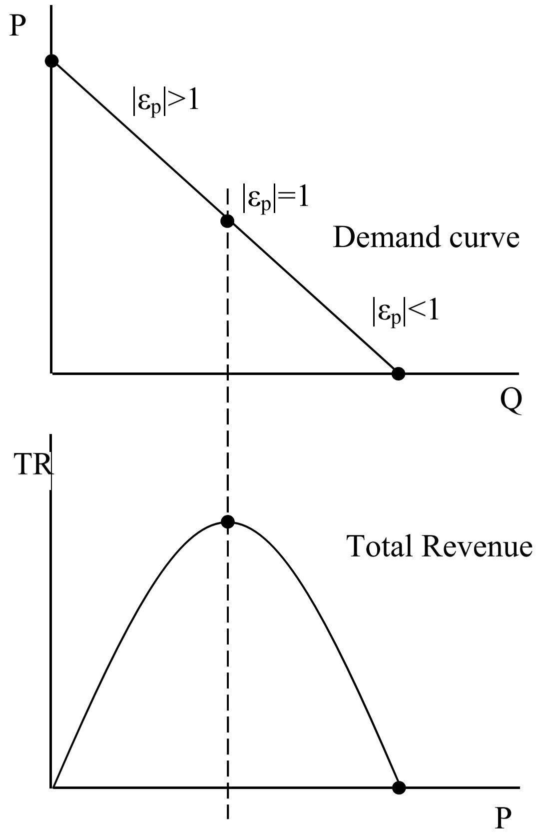

Figure 2.2. Linear demand curve and total revenue curve.

As can be seen, a linear demand function results in a nonlinear total revenue function. More precisely, the functional relationship between total revenue and quantity demanded seen here is expressed as a quadratic equation. Basically, this particular functional relationship is obtained whenever the independent variable is raised to the second power and to the first power. Graphically, quadratic equations are easily recognized by their parabolic shape.

The maximal point on TR curve is found by solving an equation

$$\frac{dTR}{dP} = b + 2kP = 0. \quad (2.35)$$

We can now present the mathematical formulas of optimal values of revenue maximization for demand (D) function of linear demand curve.

The optimal price is:

$$P_{op} = -\frac{b}{2k} \ . \quad (2.36)$$

The optimal demand quantity is:

$$D_{op} = \frac{b}{2} \, . \quad (2.37)$$

The maximized total revenue for optimal price and optimal demand quantity is:

$$TR_{max} = -\frac{b^2}{4k} \, . \quad (2.38)$$

Here are the mathematical formulas for optimal values of revenue maximization for inverse function of demand (D) function of linear demand curve.

The optimal price is:

$$P_{op} = \frac{b_1}{2} \, . \quad (2.39)$$

The optimal demand quantity is:

$$D_{op} = -\frac{b_1}{2k_1} \, . \quad (2.40)$$

The maximized total revenue for optimal price and demand quantity is:

$$TR_{max} = -\frac{b_1^2}{4k_1} \, . \quad (2.41)$$

The total cost function for the linear demand curve is:

$$C_T = C_F + C_V D, \quad (2.42)$$

where C_F – fixed cost, C_V – variable cost.

Now we can calculate the maximum profit for the linear demand curve. Profit is simply total revenue minus total cost.

$\text{Profit} = b_1D + k_1D^2 - C_F - C_VD$ (2.43)

Setting the profit function to zero we can obtain break-even points:

$k_1D^2 + (b_1-C_V)D - C_F = 0$

$$D_{1,2} = \frac{-(b_1-C_V)\pm\sqrt{(b_1-C_V)^2+4k_1C_F}}{2k_1}$$

Setting the marginal revenue profit function to zero we can obtain optimal demand quantities for profit maximization:

$$\frac{dP}{dD} = 2k_1D + b_1 - C_V = 0. \quad (2.44)$$

Thus the optimal price and demand values for the profit maximization for the linear demand curve are as follows.

The optimal price is

$$P_{op} = \frac{b_1+C_V}{2}, \quad (2.45)$$

the optimal demand quantity, therefore is

$$D_{op} = -\frac{b_1-C_V}{2k_1}, \quad (2.46)$$

And the maximized profit is

$$Profit = -\frac{b_1^2-C_v}{4k_1}. \quad (2.47)$$

2.6.3. Optimal values for nonlinear demand curve

So far it has been assumed that the price-demand relationship is linear. This is not because a linear relationship is most common in practice, but because the analysis and interpretation of such relationships is the easiest to understand. In reality the demand relationship may take a number of mathematical forms, and a particularly common form is a power form.

Although the relationships among many economic variables can be satisfactorily represented using a linear regression model, situations do occur in which a nonlinear model is clearly required to portray the relationship adequately. Various models are available to deal with these situations. These models include the semi-logarithmic transformation, the double-log transformation, reciprocal transformation, and polynomial transformations. The discussion of these transformations is beyond this study. We are mentioning them here, in order to give some sense of the nonlinear forms used for the demand curves.

The nonlinear demand curve can take the form quadratic equation.

$$D= b + kP + lP^2 \tag{2.48}$$

Clearly, that the total revenue function for the demand function in (2.48) has a cubic form:

$$TR = bP + kP^2 + lP^3 \tag{2.49}$$

The total revenue maximum point will require the solution of the following equation:

$$\frac{dTR}{dP} = b + 2kP + 3lP^2 = 0. \tag{2.50}$$

The equation (2.50) will have several roots.

It is obvious that obtaining the optimal price and demand quantity for revenue and profit maximization will require strong calculational skills.

Figure 2.3. Nonlinear demand for quadratic equation

An example of a nonlinear demand curve is shown in Figure 2.3. The corresponding curve, total revenue which presents at least two critical points is shown in Figure 2.4. Observe that total revenue curve starts from zero and increases as the quantity of demand decreases for the corresponding increasing price levels. At some point for the higher prices, total revenue stops increasing and levels up. For further price increase it starts the decreasing trend. However, it does not hit zero level; instead at further price increase it shows increasing behavior again. This second increasing behavior suggests that there is another maxima point on the total revenue curve. We cannot tell, if it is higher or lower in height than the first maximum point, but we can tell that there is some more story about the total revenue curve to be discovered.

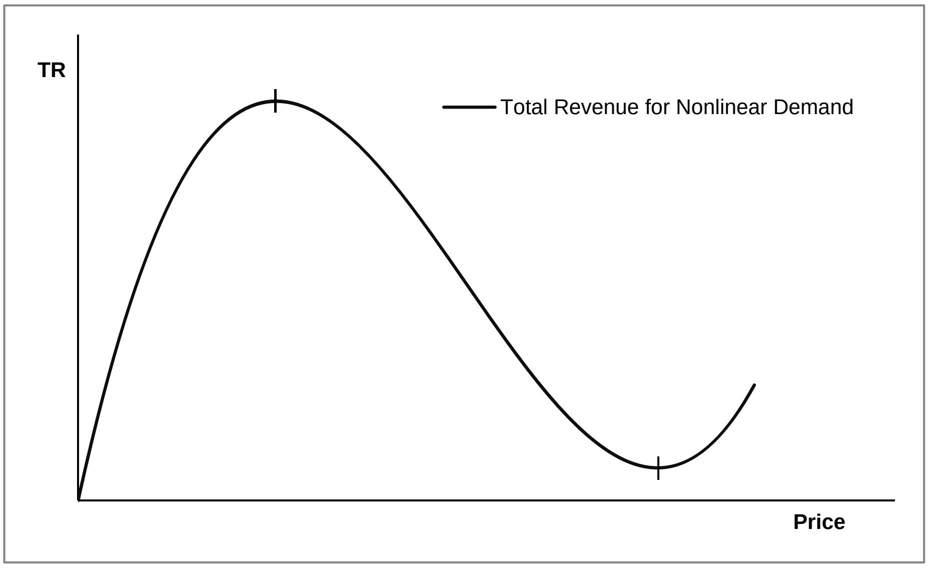

Figure 2.4. Total revenue for nonlinear demand function

2.6.4. Optimal values of integrated demand model

In section 2.2 we have introduced new model of demand – price relationship based on the assumption that market demand is a sum of several elementary demands. It implies that market demand has a complicated nonlinear character. The latter leads to the possibility of several maximums of total revenue curve and their nontrivial complicated distribution along the price values.

In this section we consider a general theoretical concept of determination and analysis of above mentioned maximums and locations along the price values.

Let us find maximums of the following total revenue (TR) functions.

$$TR = \left(\sum_{i=1}^{r} d_i\left(1 - \frac{z}{x_i}\right)\right) z = z\sum_{i=1}^{r} d_i - z^2 \sum_{i=1}^{r} \frac{d_i}{x_i}. \tag{2.51}$$

Differentiation of *TR* with respect to *z* gives following:

$$\frac{dTR}{dz} = \sum_{i=1}^{r} d_i - 2z\sum_{i=1}^{r} \frac{d_i}{x_i} = 0. \tag{2.52}$$

Solution of this equation gives points where *TR* attains maximum.

$$z_{op} = \frac{\sum_{i=1}^{r} d_i}{2\sum_{i=1}^{r} \frac{d_i}{x_i}} < 1\,, \tag{2.53}$$

since $\frac{d_i}{x_i} > d_i$.

It is straightforward to obtain corresponding value of maximum of total revenue by substituting (2.53) into (2.51).

$$TR_{max} = \frac{(\sum_{i=1}^{r} d_i)^2}{4\sum_{i=1}^{r} \frac{d_i}{x_i}}. \tag{2.54}$$

Note that in all above mentioned expressions we used complete sums connected with all backup points (i=0, …, r). Clearly, if one starts to calculate sums in (2.53) and (2.54) for different initial value of summation index, then the different values of z_{op} and TR_{max} will be obtained:

$$z_{op_k} = \frac{\sum_{i=k}^{r} d_i}{2\sum_{i=k}^{r} \frac{d_i}{x_i}}, \tag{2.55}$$

$$TR_{max_k} = \frac{(\sum_{i=k}^{r} d_i)^2}{4\sum_{i=k}^{r} \frac{d_i}{x_i}}, \tag{2.56}$$

or

$$TR_{max_k} = z_{op_k} \frac{\sum_{i=k}^{r} d_i}{2}. \tag{2.57}$$

It means that total revenue generated by integrated demand might have *r* optimal prices and maximums of total revenues. More detailed analysis presented below shows that the number of optimal prices and maximums of demand may vary from 1 to *r*. The latter depends on the parameters of elementary demands.

General shape of integrated total revenue curve is shown in Figure 2.5. Interesting to note that the total revenue curve of integrated demand, being piecewise differentiable, has no minimums, but only maximums.

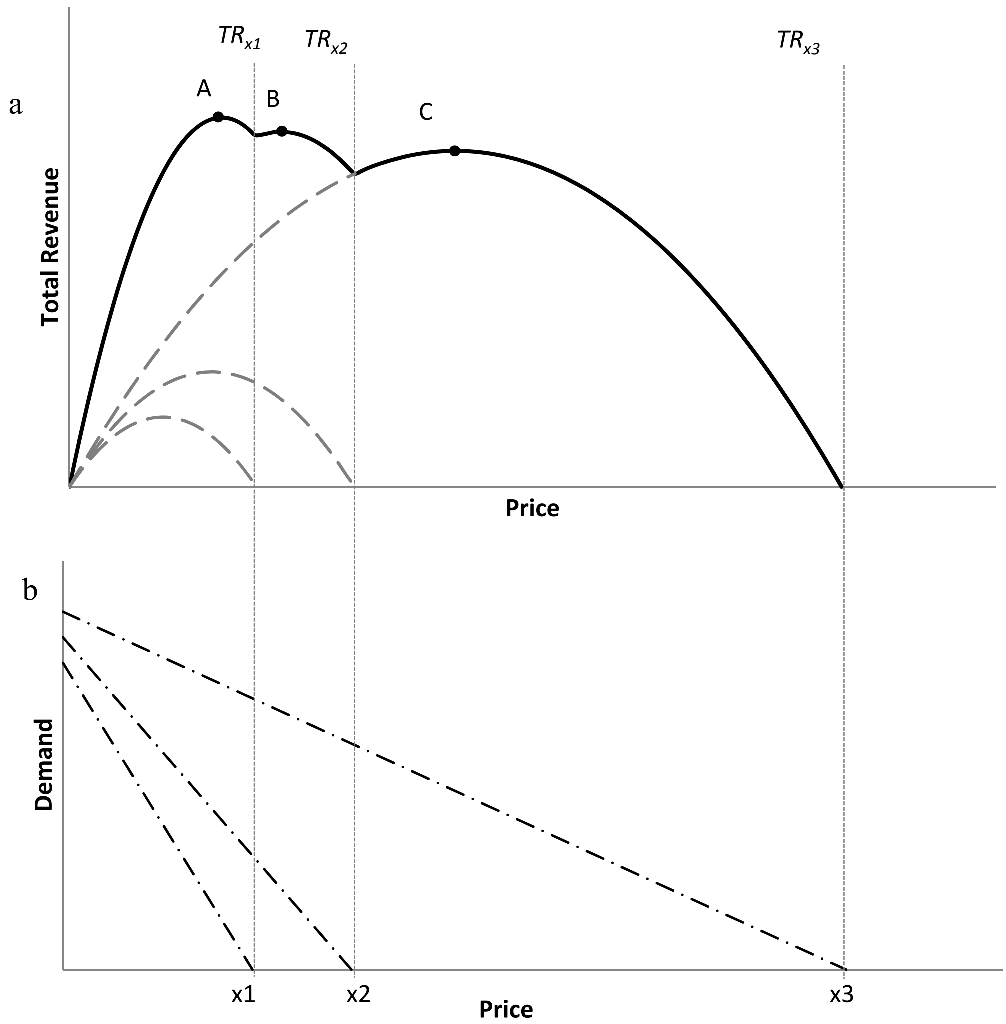

Figure 2.5. Integrated total revenue curve, *TR*s of elementary demands, and elementary demand lines:

a) **Integrated total revenue and elementary demands' revenue parabolas;**
b) **Elementary demand lines**

Total revenue curve in backup points x_i, being continuous, has no derivatives. So, these points can be considered as minimums.

The first maximum, indicated by capital letter A in figure 2.5,

$$TR_{max_1} = \frac{\left(\sum_{i=1}^{r} d_i\right)^2}{4\sum_{i=1}^{r}\frac{d_i}{x_i}}$$

is formed by means of involvement of all demand – price straight lines and their corresponding total revenue parabolas, whereas the second one (capital letter B in figure 2.5) is generated by last two lines and so on.

It is important to note, that some maximum values of total revenue may not appear in the integrated total revenue curve.

Let us denote the integrated total revenue in the backup points x_i as TR_{xi}. It is clear, that total revenue values in backup points x_i are

$$TR_{x_i} = x_k \sum_{i=k}^{r} d_i - x_k^2 \sum_{i=k}^{r} \frac{d_i}{x_i}. \tag{2.58}$$

Then, it is easy to understand that maximum value of total revenue will be explicitly shown on integrated total revenue curve (we call such values explicit values of total revenue) if

$$TR_{max_k} \geq TR_{x_k} \tag{2.59}$$

Inequality (2.59) implies simple criterion to detect whether the given maximum of integrated total revenue is explicit or implicit.

From (2.59) and (2.57) we get in case of equality

$$z_{op_k} \cdot \frac{\sum_{i=k}^{r} d_i}{2} = x_k \sum_{i=k}^{r} d_i - x_k^2 \sum_{i=k}^{r} \frac{d_i}{x_i}. \tag{2.60}$$

The equation (2.60) can be transformed to second order algebraic equation:

$$z_{op_k}^2 - 2x_k z_{op} + x_k^2 = 0, \tag{2.61}$$

which immediately gives the following unique root:

$$z_{op_k} = x_k \text{ , } (k = 1, 2, \ldots, \mathrm{r}). \tag{2.62}$$

The last criterion (2.62) shows that if z_{op_k} , optimal point of integrated total revenue, is:

1) less than x_k, then optimal value of integrated total revenue is implicit (it does not fall in integrated total revenue curve);
2) equal to x_k , then corresponding maximum of integrated total revenue falls in intersection of adjacent parabolas;
3) greater than x_k, then corresponding maximum of integrated total revenue curve is explicit and it can be pointed out in integrated total revenue curve.

In chapter 3 we shall analyze all these cases in details.

CHAPTER 3. THE COMPUTATIONAL POWER OF INTEGRATED DEMAND MODEL

3.1. Description of the Model

From Chapter 2 it is clear that one cannot directly apply conventional regression analysis for the problem of estimation of parameters of the elementary smoothed demands and integrated smoothed demands. As there are several elementary components one should use multiple regression analysis techniques, but interpretation of final results should be done in terms of one independent variable.

The first step is to form *m* x *n* matrix of *m* independent variables of price, where *m* is the number of anticipated elementary demands, defined by backup-prices and observed data. We have created special function in MATLAB programming language. Input parameters of the function are: *n* - number of observations of demand; *m* – number of backup prices. We assume that length intervals between any two backup prices are equal. The output of the function is *m* x *n* matrix X of variable prices corresponding to backup prices. Note that values of the price variables greater than backup prices are equal to zero. So the matrix X consists of values between 1 and 0 for price variables less than backup prices and all 0 values for price variables greater than them. Each column of the matrix should be interpreted as independent variable, so there are *m* independent variables of price. This approach is very similar to the methodology of regression of dummy variables introduced by Suits (1957).

Difference of elaborated approach from the conventional regression analysis is that after estimation of all necessary parameters, one will interpret the final result in terms of only one variable. As it actually is in our case, there is only one variable, and it is a price. Thus we are interested in detecting a relationship between demand and this only price. The developed computational function in MATLAB programming language is shown in *Appendix 1*.

After forming the matrix of input variables conventional linear multiple regression method should be applied to determine values of d_i (2.9). To illustrate all possibility of the elaborated approach, we consider various examples of the different data structures in following sections. In order to explore the significance of the model output results we discuss only *F*-statistics. The *F*-statistic is a measure of the statistical significance of the relationship between the dependent variable and the independent variables taken as a group. It is the ratio of explained variance to unexplained variance. If there is just one independent variable the *F*-statistic gives the same result as the *t*-statistic.

3.2. Dummy Variables and Their Usage

Marshall (1998) states that in quantitative data analysis, researchers are sometimes interested in the implications of non-interval level variables for a dependent variable, as for example in the case of the relationship between sex and income. Regression analysis require the collected data to be scaled at interval level, however it is possible to create appropriate so-called dummy variables of non-interval variables that can be included in a multiple regression. The following example of provided by Marshall (1998) helps understanding the usage of dummy variables. Independent variable with only two categories would involve coding for one category as 1 and other category coding as 2, as for the coding sex would be coding men as 1 and women as 0. In case the independent variable contains more than two (*n*) categories, creating a dummy variable would involve coding $n - 1$ dummy variables. For example, the independent variable "social class" is containing four (n=4) categories "upper", "middle", "working", and "none". Three dummy variables (n-1) should be created in order to include these in a multiple regression analysis: first variable "upper" is coded 1 or "not upper" is coded 0; second variable "middle" is coded 1 or "not middle" is coded 0; third variable "working" is coded 1 or "not working" is coded 0. The combination 000 can describe the fourth category represented by these three dummy variables. Consequently, the "social class" independent variable's categories have been coded by a unique combination of zeros or ones. These coding will help to indicate the presence or absence of each categories. The resulting regression coefficients from usage of dummy variables are regarded as if they were based on variables measured at the interval level.

Dummy variables are independent variables which take the value of either 0 or 1. A dummy variable is used to numerically express a qualitative fact or a logical proposition. Caravaglia and Sharma (2000) conclude that there should be another way to represent qualitative concepts such as season, male or female, smoker or non-smoker, etc. in order for for many models to make sense. They support their conclusion by providing an example of a model that is used to estimate demand for electricity in a geographical area. The model might include the average temperature, the average number of daylight hours, the total number of structure square feet, numbers of businesses, numbers of residences, and so forth. However, the model might be more useful, if it could produce appropriate results for each month or each season. Using the number of the month would be inappropriate because the demand for electricity is going to be very different between different months as well as the winter occurs during the different months on different polar hemispheres.

According to Caravaglia and Sharma (2000) coding a dummy variable by 0 will cause its coefficient to disappear from the equation and by 1 will cause its coefficient to function as a supplemental intercept, because of the identity property of multiplication by 1. In a linear regression model the definition of the subsets of observations that have different intercepts and sloped may require creation of separate models; however the introduction of the dummy variables will eliminate this necessity. The calculation of the odds ratios and their easy interpretation as well as the increase in significance and stability of the coefficients of the logistic regression functions is obtained by encoding all of the independent variables as dummy variables.

Representation of the information in the form of the dummy variables makes it easier to turn the model into decision tool, rather than being beneficial only to statistical analysis. The risk manager dealing with the applications for the credit should assign the limits of the credit per each business. One of the significant criteria in assessing the exposable risk is the age of the business. It is not wise for the risk manager to assign a different credit limit for each year in business having that some businesses of several hundred year old. Therefore, Caravaglia and Sharma (2000) conclude that bivariate analysis of the relationship between age of business and default usually yields a small number of groups that are far more statistically significant than each year evaluated separately.

Synonyms for dummy variables are design variables (Hosmer and Lemeshow, 1989), boolean indicators, and proxies (Kennedy, 1989). Related concepts are binning (Tukey, 1977) or ranking, because belonging to a bin or rank could be formulated into a dummy variable. Bins or ranks can also function as sets and dummy variables can represent non-probabilistic set membership. Set theory is usually explained in texts on computer science or symbolic logic. See for more details (Arbib, et. al., 1981) or (MacLane, 1986).

Caravaglia and Sharma (2000) remark that dummy variables based on set membership can help when there are too few observations, and thus, degrees of freedom, to have a dummy variable for every category or some categories are too rare to be statistically significant.

In the analysis of data dummy variables play an important role, indifferently if the data under analysis are real-valued variables, categorical data, or analog signals. In selecting a modeling methodology representation of all the variables, independent and dependent, by dummy variables provides a high degree of flexibility. Beyond the benefit of flexibility, the probabilistic reasoning, information theory, set relations, and symbolic logic have interpretations in elementary statistics, such as mean and standard deviation, by dummy variables.

It is obvious that dummy variables can represent any highly complex information structures, independent of the traditional or experimental analytical technique. As Caravaglia and Sharma (2000) say, there are no hard boundaries between the relationships of dummy variables in quantitative analysis, sets and logic, and the computer science concept of data representation in bits. Generally, in order to have a resulting application or model to be easy to implement, use, and interpret, the dummy variables should be intelligently used.

3.3. Datasets of Observations and Integrated Demand Model

3.3.1. Case 1: Polynomial shape dataset of 21 observations

Polynomial shape is presented by the set of 21 observations of data generated by the random number distribution. The developed integrated market regression model is applied to get the regression statistics for this dataset. We have used three, five, and seven backup points to cross compare the results obtained by the proposed regression model as well as to compare those results with the result of non-linear polynomial regression. The non-linear polynomial regression is assumed to be appropriate for the dataset under consideration and obviously the cloud of observed data is not well suited for linear regression. We are interested in the overall significance of the regression model, thus we will discuss only F statistics with its critical value.

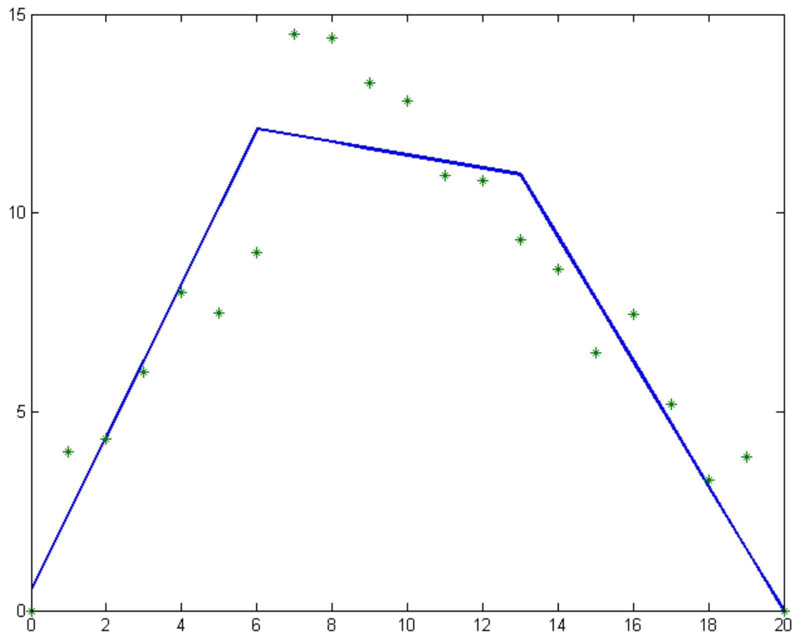

Figure 3.1. Integrated demand model for three backup points.

3.3.1.1. Integrated demand model with three backup points

In this case we have used three backup points to obtain the regression statistics output presented in Table 3.1 and the curve presented in Figure 3.1 correspondingly.

Table 3.1

ANOVA - case of three backup points

Source	df	SS	MS	F
Regr	3.0000	314.5517	104.8506	37.2413
Resid	17.0000	47.8624	2.8154	
Total	20.0000	362.4141		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.19) than the observed value of F (37.24), which means that integrated demand model with three backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 3 backup points does explain a significant proportion of the variation in the sample.

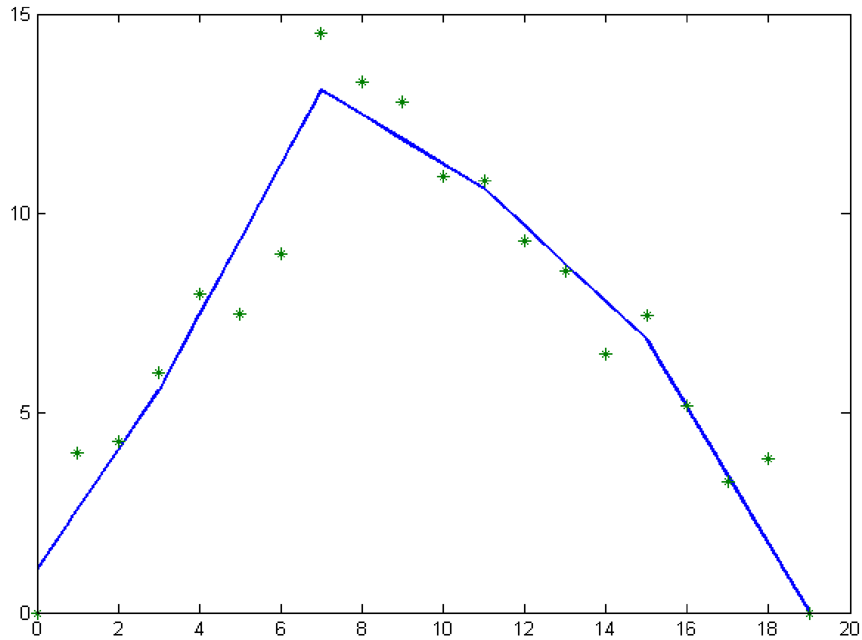

Figure 3.2. Integrated demand model for five backup points.

3.3.1.2. Integrated demand model with five backup points

In this case we have used five backup points to obtain the regression values presented in Table 3.2 and the curve presented in Figure 3.2 correspondingly.

Table 3.2

ANOVA - case of five backup points

Source	df	SS	MS	F
Regr	5.0000	326.1095	65.2219	35.7837
Resid	14.0000	25.5174	1.8227	
Total	19.0000	351.6269		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (2.96) than the observed value of *F* (35.78), which means that integrated demand model with five backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 5 backup points does explain a significant proportion of the variation in the sample.

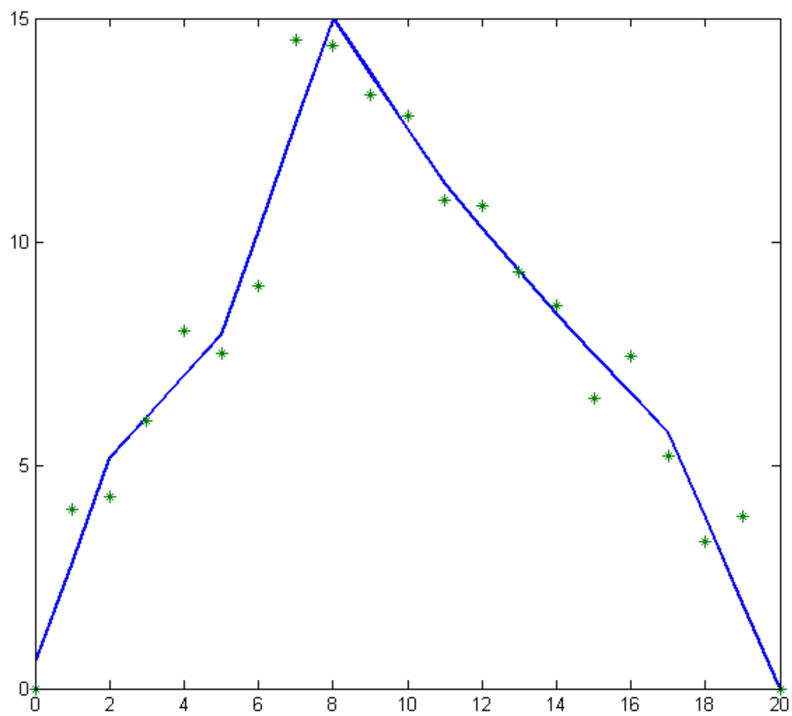

Figure 3.3. Integrated demand model for seven backup points.

3.3.1.3. *Integrated demand model with seven backup points*

In this case we have used seven backup points to obtain the regression values presented in Table 3.3 and the curve presented in Figure 3.3 correspondingly.

Table 3.3

ANOVA - case of seven backup points

Source	df	SS	MS	F
Regr	7.0000	347.4576	49.6368	43.1435
Resid	13.0000	14.9566	1.1505	
Total	20.0000	362.4141		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (2.83) than the observed value of *F* (43.14), which means that integrated demand model with seven backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 7 backup points does explain a significant proportion of the variation in the sample.

3.3.1.4. *Polynomial regression results for the dataset of Case 1*

The data set under consideration has a polynomial shape so that it is not a choice to apply linear analysis which may require much calculation.

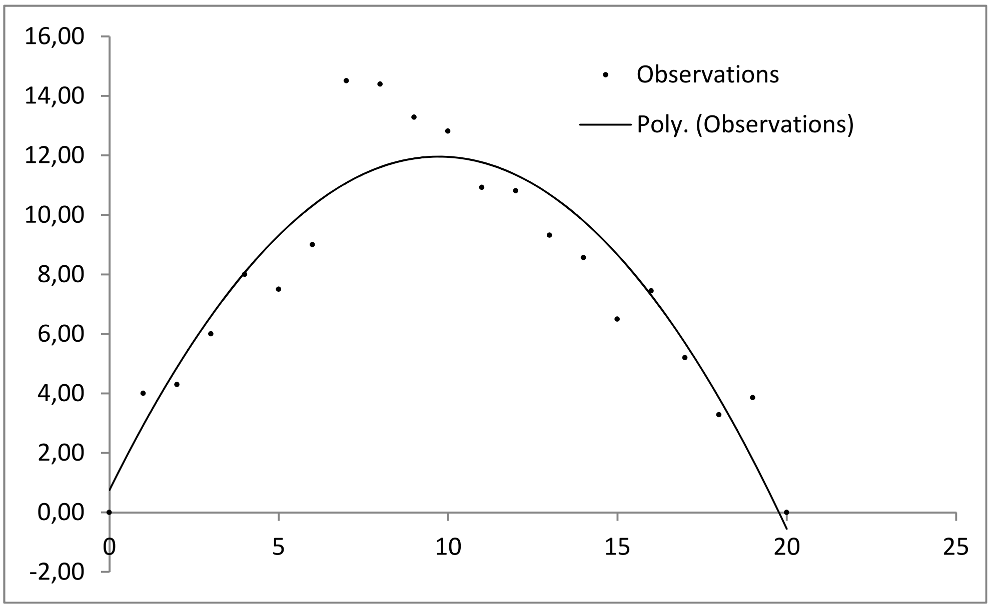

Figure 3.4. Polynomial trend line of 2nd order

Thus we have applied nonlinear polynomial regression analysis. Both second and third order polynomial regression statistics output with the critical values of F statistics are presented below in Figure 3.4 and Figure 3.5 and in Table 3.4 and Table 3.5 correspondingly.

Table 3.4

ANOVA - Polynomial regression of 2nd order for case 1

Source	df	SS	MS	F
Regression	2	600,5728	300,2864	735,7080
Residual	18	7,3469	0,4082	
Total	20	607,9197		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.55) than the observed value of F (735.70). Polynomial regression of second order explains a significant proportion of the variation in the given dataset.

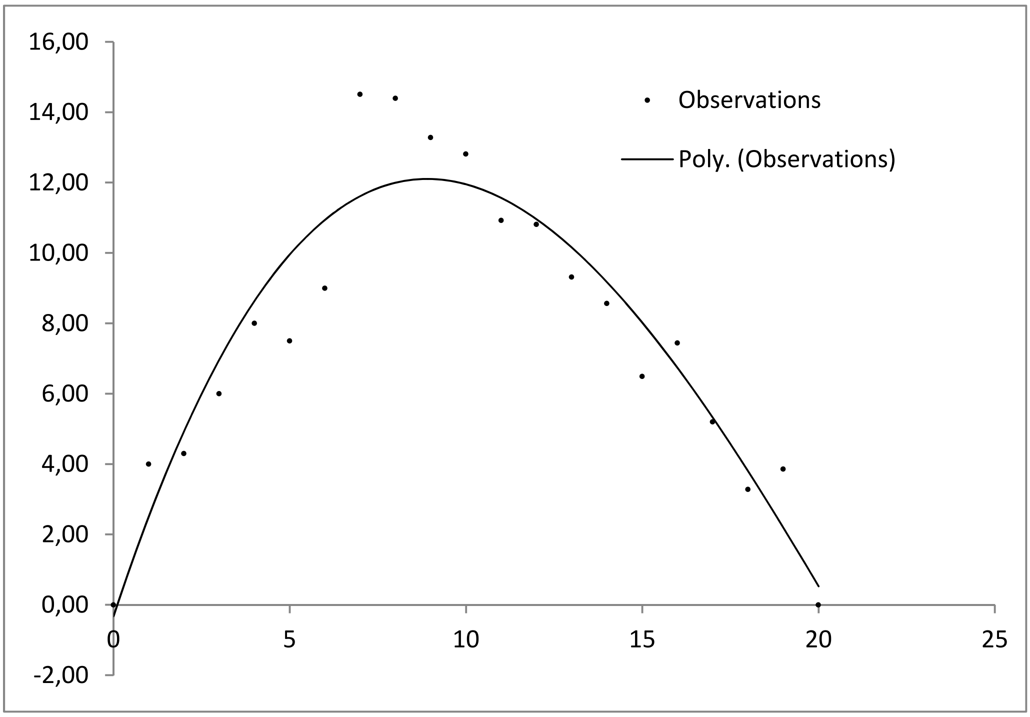

Figure 3.5. Polynomial trend line of 3rd order

Table 3.5

ANOVA - Polynomial regression of 3rd order for case 1

Source	df	SS	MS	F
Regression	3	600,5728842	200,1909614	463,225741
Residual	17	7,346842031	0,432167178	
Total	20	607,9197262		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.19) than the observed value of F (463.22). Polynomial regression of third order explains a significant proportion of the variation in the given dataset.

3.3.1.5. Discussion of results for the Case 1

The regression statistics for three, five, and seven backup points used in our integrated demand model demonstrated the significance in interpreting the observed dataset. The F statistics (37.24, 35.78, and 43.14 correspondingly for three, five, and seven backup points) were more than their critical values (3.19, 2.96, and 2.83 correspondingly for three, five, and seven backup points). The presented regression results of the integrated demand model have no doubt validated adequacy of the mentioned regression model.

Both second and third order polynomial regression statistics presented better significance in fitting the data (F statistics 735.70 and 463.22 with critical values of 3.55 and 3.19 correspondingly for second order and third order of polynomial regression) than the results of our integrated model. The second order of polynomial regression gives better interpretation of the data in means of F statistics.

The best explanation for the significant proportion of the observed dataset, from integrated market demand model results, is presented by the seven backup points' case (F value = 43.14), followed by three backup points' case (F value = 37.24), and then by five backup points' case (F value = 35.78). It is easy to see that the adequacy of all three models is extremely high, so we cannot conclude, from this point of view, which model is preferable.

If there is no a priori pure economical requirements on number of backup points (for example, one have empirical economical information which leads to the necessity of using five backup points) then it is clear that model with minimal backup points should be chosen due to obvious simplicity.

3.3.2. Case 2: Polynomial shape dataset of 21 observations

2 order polynomial shape is presented by the set of 21 observations of data. The developed integrated market regression model is applied to get the regression values for this data set. We have used three, five, and seven backup points to compare outcomes across the results obtained by the proposed regression model as well as to compare those results with the result of non-linear polynomial regression. The non-linear polynomial regression is assumed to be appropriate for the data set under consideration and obviously the cloud of observed data is not well suited for linear regression. We are interested in the overall significance of the regression model, thus we will discuss only *F* statistics with its critical value.

3.3.2.1. Integrated market demand model with three backup points

In the Figure 3.6 and Table 3.6 are presented the graphical representation and ANOVA results for the developed regression model for the three backup points. The curve in Figure 3.6 consists of three parts passing through the data set continuously, so that the curve is fully formed.

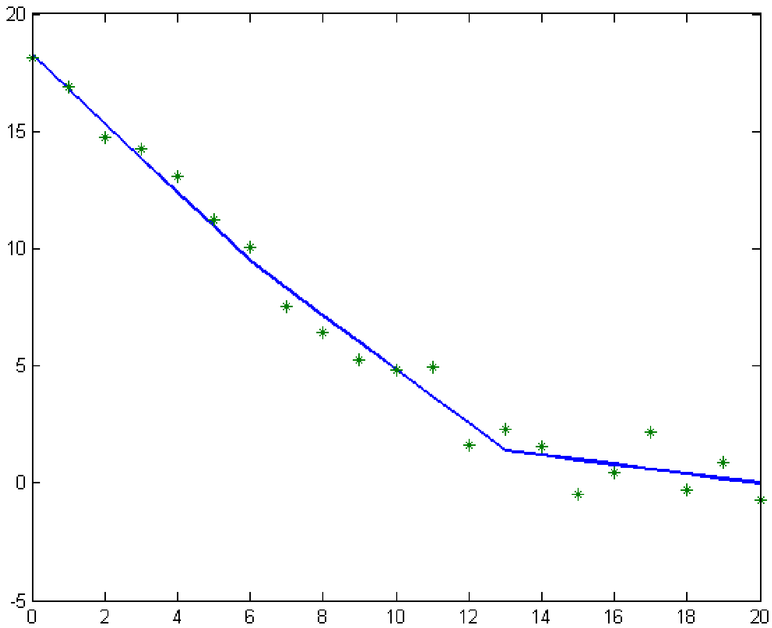

Figure 3.6. Integrated demand model for three backup points.

Table 3.6 *ANOVA – case 2 of three backup points*				
Source	**df**	**SS**	**MS**	**F**
Regr	3.0000	1043.6601	347.8867	470.2587
Resid	17.0000	12.5762	0.7398	
Total	20.0000	1056.2364		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.19) than the observed value of F (470.26), which means that integrated demand model with three backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 3 backup points does explain a significant proportion of the variation in the sample.

3.3.2.2. Integrated market demand model with five backup points

In the Figure 3.7 and Table 3.7 are presented the graphical representation and ANOVA results for the developed regression model for the five backup points.

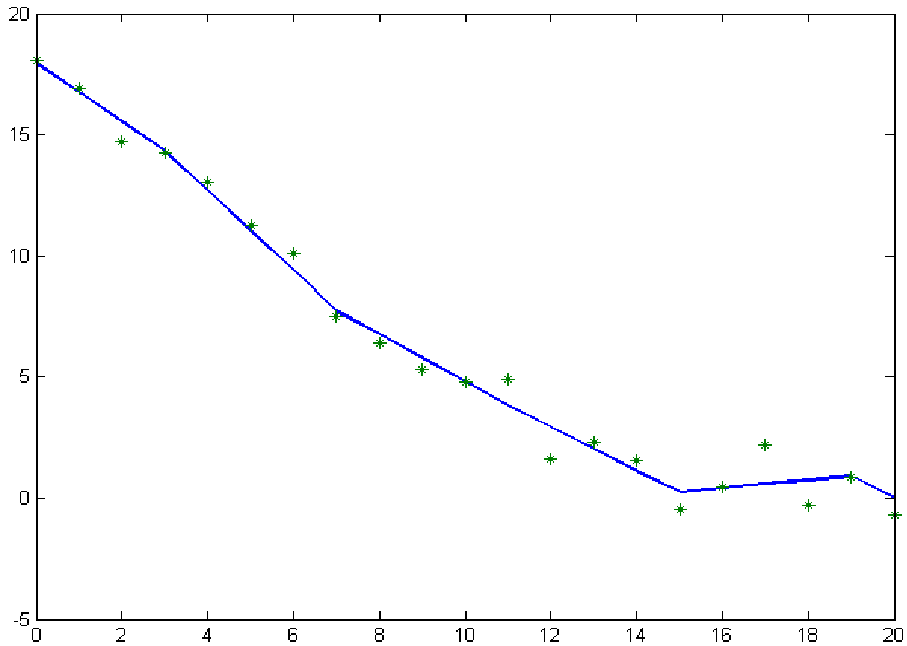

Figure 3.7. Integrated demand model for five backup points.

The curve in fig. 3.7 consists of five parts passing through the data set continuously, so that the curve is fully formed.

Table 3.7

ANOVA - case 2 of five backup points

Source	df	SS	MS	F
Regr	5.0000	960.5530	192.1106	792.7523
Resid	14.0000	3.3927	0.2423	
Total	19.0000	963.9457		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (2.96) than the observed value of F (792.75), which means that integrated demand model with five backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 5 backup points does explain a significant proportion of the variation in the sample.

3.3.2.3. Integrated market demand model with seven backup points

In the Figure 3.8 and Table 3.8 are presented the graphical representation and ANOVA results for the developed regression model for the seven backup points.

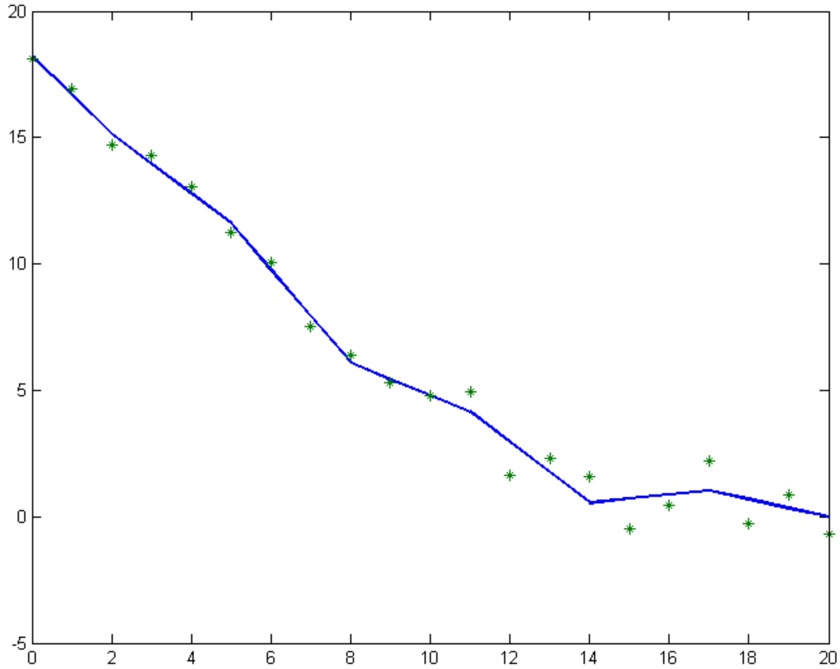

Figure 3.8. Integrated regression model for seven backup points.

The curve in fig. 3.8 consists of seven parts passing through the data set continuously, so that the curve is fully formed.

Table 3.8

ANOVA - case 2 of seven backup points

Source	df	SS	MS	F
Regr	7.0000	1055.1293	150.7328	1770.0901
Resid	13.0000	1.1070	0.0852	
Total	20.0000	1056.2364		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (2.83) than the observed value of F (1770.09), which means that integrated demand model with seven backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 7 backup points does explain a significant proportion of the variation in the sample.

3.3.2.4. Polynomial regression results for the dataset of Case 2

The data set under consideration has a polynomial shape so that it is not a choice to apply linear or multiple linear regression analysis which may require much calculations. Thus we have applied nonlinear polynomial regression analysis.

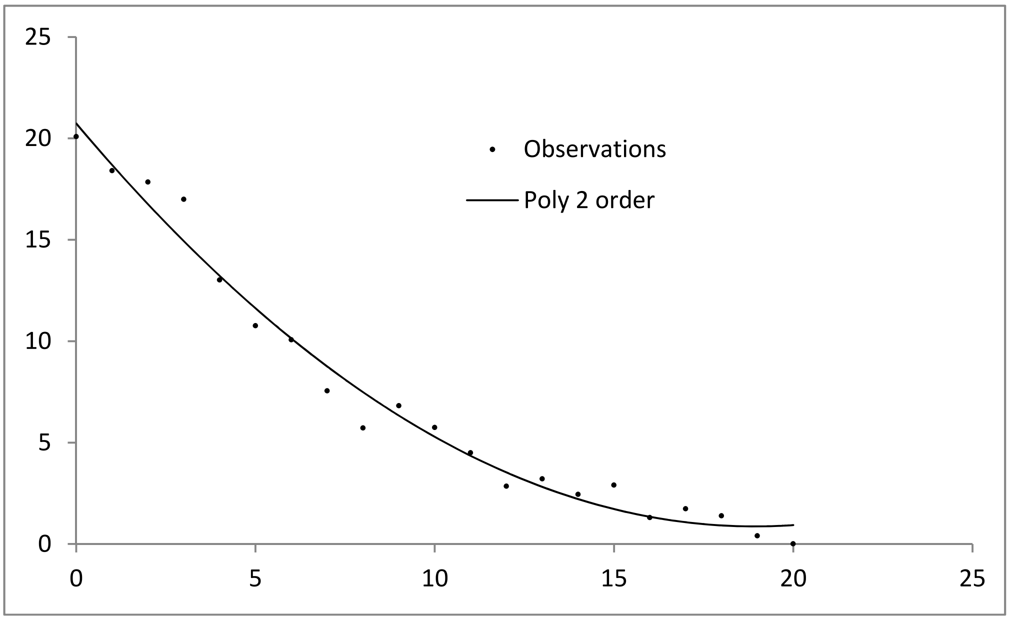

Figure 3.9. Polynomial trend line of 2nd order

Both second and third order polynomial regression statistics output with the critical values of *F* statistics are presented below in Figure 3.9 and Figure 3.10 and in Table 3.9 and Table 3.10 correspondingly.

Table 3.9

ANOVA - Polynomial regression of 2nd order for case 2

Source	df	SS	MS	F
Regression	2	1951,4293	975,7147	344,9574
Residual	18	50,9131	2,8285	
Total	20	2002,3424		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (3.55) than the observed value of *F* (344.96). Polynomial regression of second order explains a significant proportion of the variation in the given dataset.

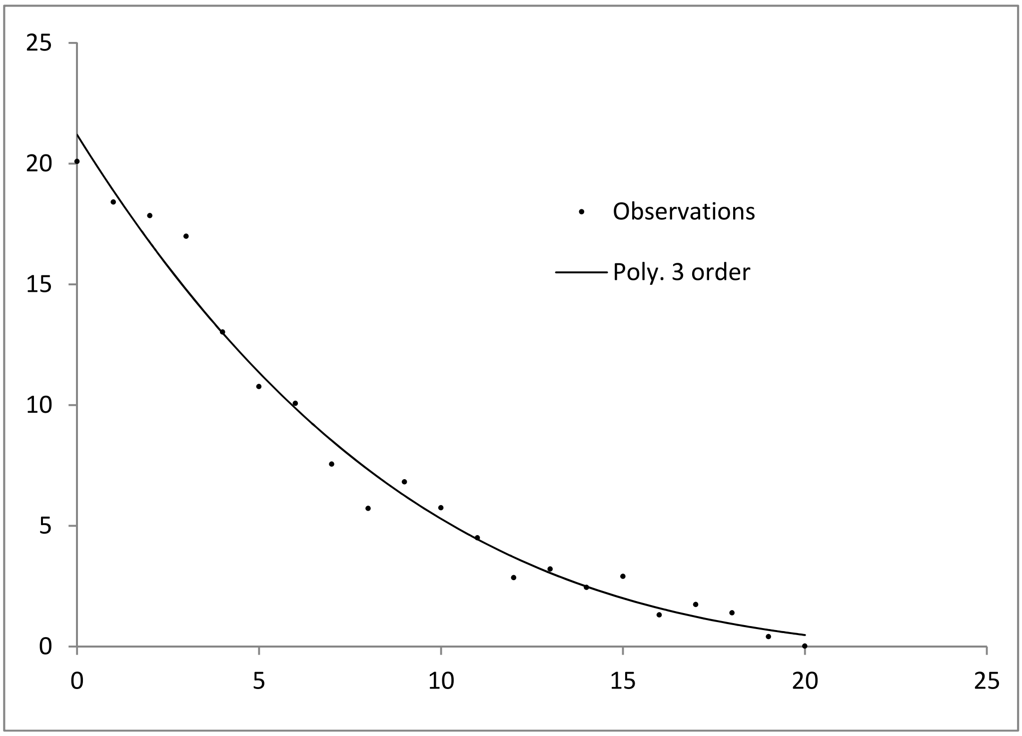

Figure 3.10. Polynomial trend line of 3rd order

Table 3.10

ANOVA - Polynomial regression of 3rd order for case 2

Source	df	SS	MS	F
Regression	3	1964,8564	654,9521	297,0225
Residual	17	37,4860	2,2051	
Total	20	2002,3424		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.19) than the observed value of F (297.02). Polynomial regression of third order explains a significant proportion of the variation in the given dataset.

3.3.2.5. Discussion of results

The regression statistics for three, five, and seven backup points used in our integrated demand model demonstrates the significance in interpreting the given polynomial shape of dataset. The F statistics (470.26, 792.75, and 1770.09 correspondingly for three, five, and seven backup points) were more than their critical values (3.19, 2.96, and 2.83 correspondingly for three, five, and seven backup points). The presented regression results of the integrated demand model has no doubt validate adequacy of the mentioned regression model.

Both second and third order polynomial regression statistics presented significance in fitting the data (F statistics 344.96 and 297.02 with critical values of 3.55 and 3.19 correspondingly for second order and third order of polynomial regression). The second order of polynomial regression gives better interpretation of the data in means of F statistics. However the integrated regression model showed better result in interpreting given data, than the polynomial regression.

The best explanation for the significant proportion of the observed dataset, from integrated regression model results, is presented by the seven backup points' case (F value = 1770.09), followed by five backup points' case (F value = 792.75), and then by three backup points' case (F value = 470.26). It is easy to see that the adequacy of all three models is extremely high, so we cannot conclude, from this point of view, which model is preferable.

If there is no a priori pure economical requirements on number of backup points (for example, one have empirical economical information which lets to the necessity of using five backup points) then it is clear that model with minimal backup points should be chosen due to obvious simplicity.

3.3.3. Case 3: Sideways S shape dataset of 35 observations

Sideways S shape dataset of observations is presented by the dataset of 35 observations. The developed integrated market regression model is applied to get the regression values for this data set. We have used three, five, and seven backup points to compare outcomes across the results obtained by the proposed integrated demand model as well as to compare those results with the result of non-linear polynomial regression. The non-linear polynomial regression is assumed to be appropriate for the data set under consideration and the cloud of observed data is not well suited for linear regression. We are interested in the overall significance of the regression model, thus we will discuss only *F* statistics with its critical value.

3.3.3.1. Integrated market demand model with three backup points

In the Figure 3.11 and Table 3.11 are presented the graphical representation and ANOVA results for the developed regression model for the three backup points. The curve in fig. 11 consists of three parts passing through the data set continuously, so that the curve is fully formed.

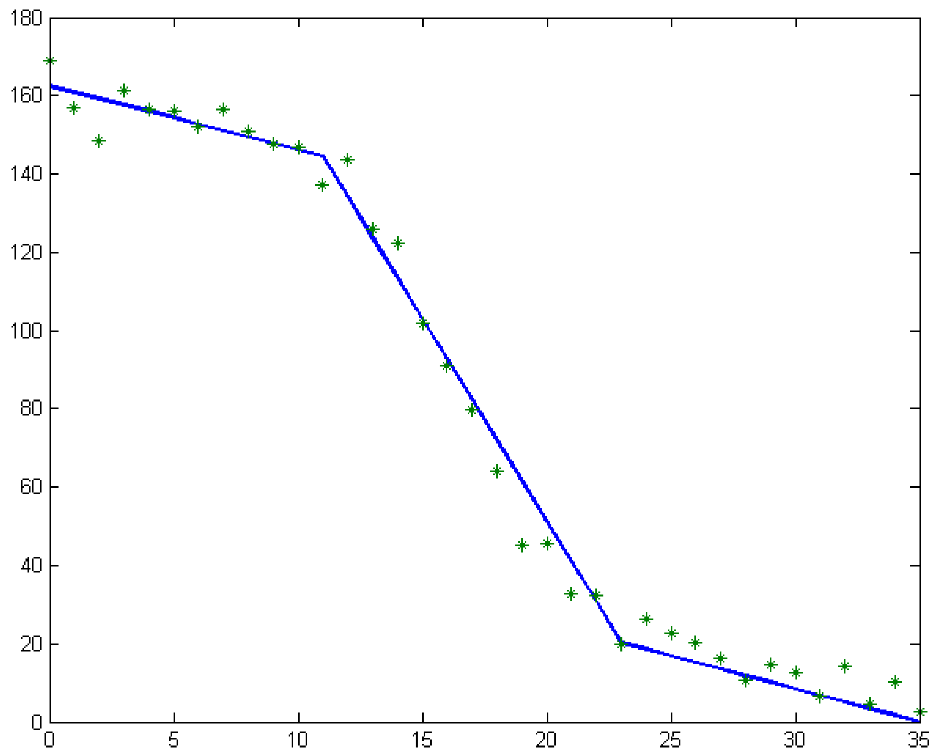

Figure 3.11. Integrated demand model for three backup points.

Table 3.11 *ANOVA - case 3 of three backup points*				
Source	**df**	**SS**	**MS**	**F**
Regr	3.0000	137075.4346	45691.8115	1375.8762
Resid	31.0000	1062.6959	33.2092	
Total	34.0000	138138.1305		

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (2.91) than the observed value of F (1375.88), which means that integrated demand model with three backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 3 backup points does explain a significant proportion of the variation in the sample.

3.3.3.2. Integrated market demand model with five backup points

In the Figure 3.12 and Table 3.12 are presented the graphical representation and ANOVA results for the developed regression model for the five backup points. The curve in fig. 12 consists of five parts passing through the data set continuously, so that the curve is fully formed.

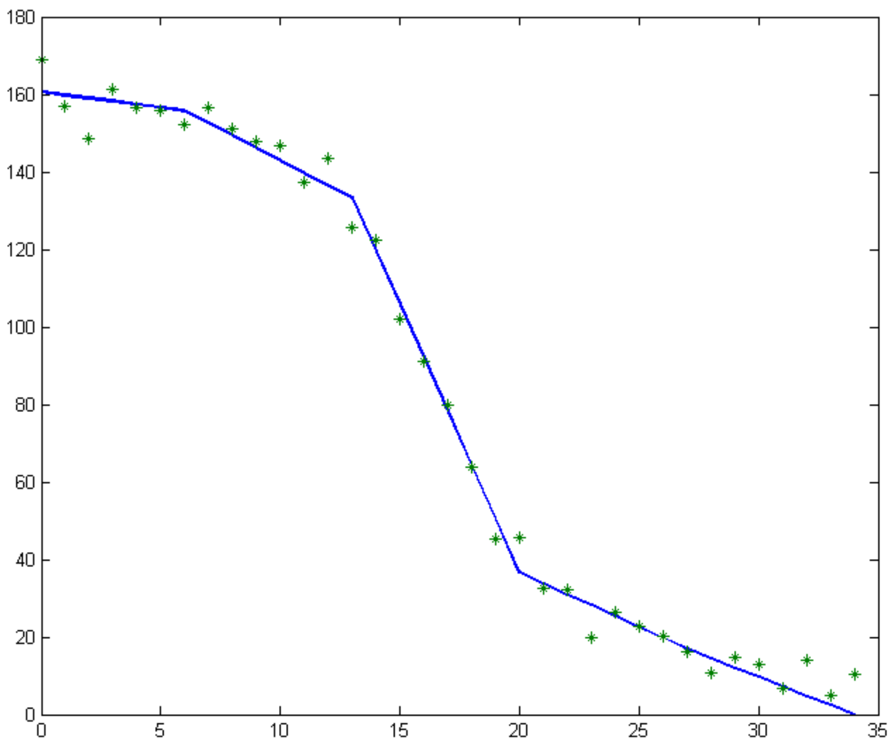

Figure 3.12. Integrated demand model for five backup points.

Table 3.12 *ANOVA - case 3 of five backup points*				
Source	**df**	**SS**	**MS**	**F**
Regr	5.0000	131089.5182	26217.9036	1181.7050
Resid	29.0000	643.4086	22.1865	
Total	34.0000	131732.9269		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (2.55) than the observed value of *F* (1181.70), which means that integrated demand model with five backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 5 backup points does explain a significant proportion of the variation in the sample.

3.3.3.3. Integrated market demand model with seven backup points

In the Figure 3.13 and Table 3.13 are presented the graphical representation and ANOVA results for the developed regression model for the seven backup points. The curve in fig. 13 consists of seven parts passing through the data set continuously, so that the curve is fully formed.

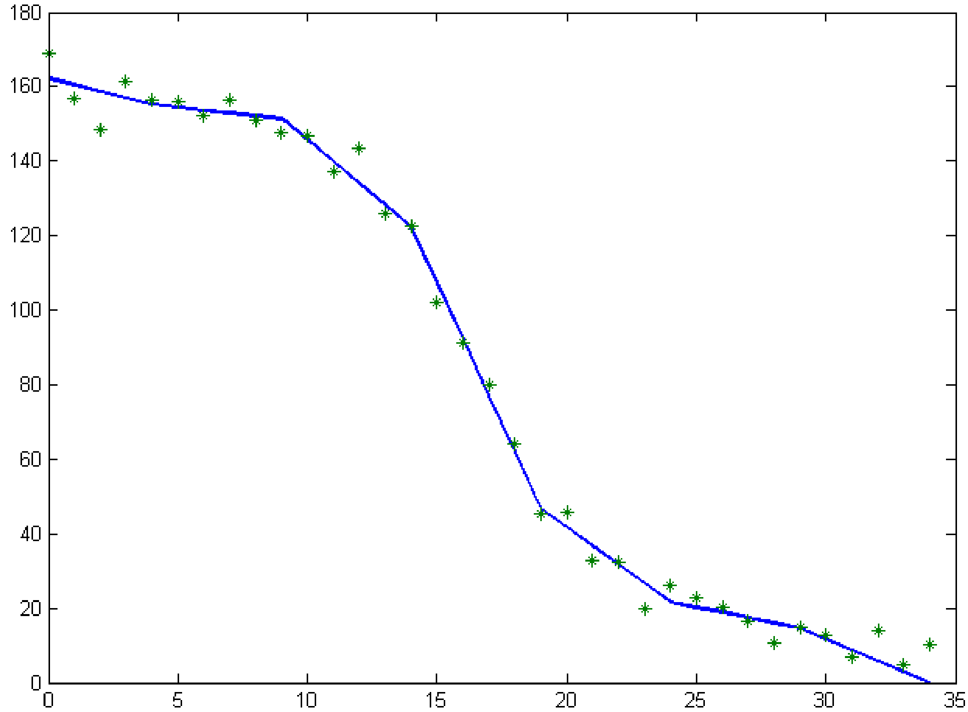

Figure 3.13. Integrated demand model for seven backup points.

Table 3.13

ANOVA – case 3 of seven backup points

Source	df	SS	MS	F
Regression	7.0000	131172.5662	18738.9380	902.9031
Residual	27.0000	560.3606	20.7541	
Total	34.0000	131732.9269		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (2.37) than the observed value of *F* (902.90), which means that integrated demand model with seven backup points adequately describes the given data by the provided means of measurement. In other words, we conclude that the integrated demand model with 7 backup points does explain a significant proportion of the variation in the sample.

3.3.3.4. Polynomial regression results for the dataset of Case 3

The data set under consideration has a Sideways S shape dataset of observations so that it is not a choice to apply linear or multiple linear regression analysis which may require much calculations.

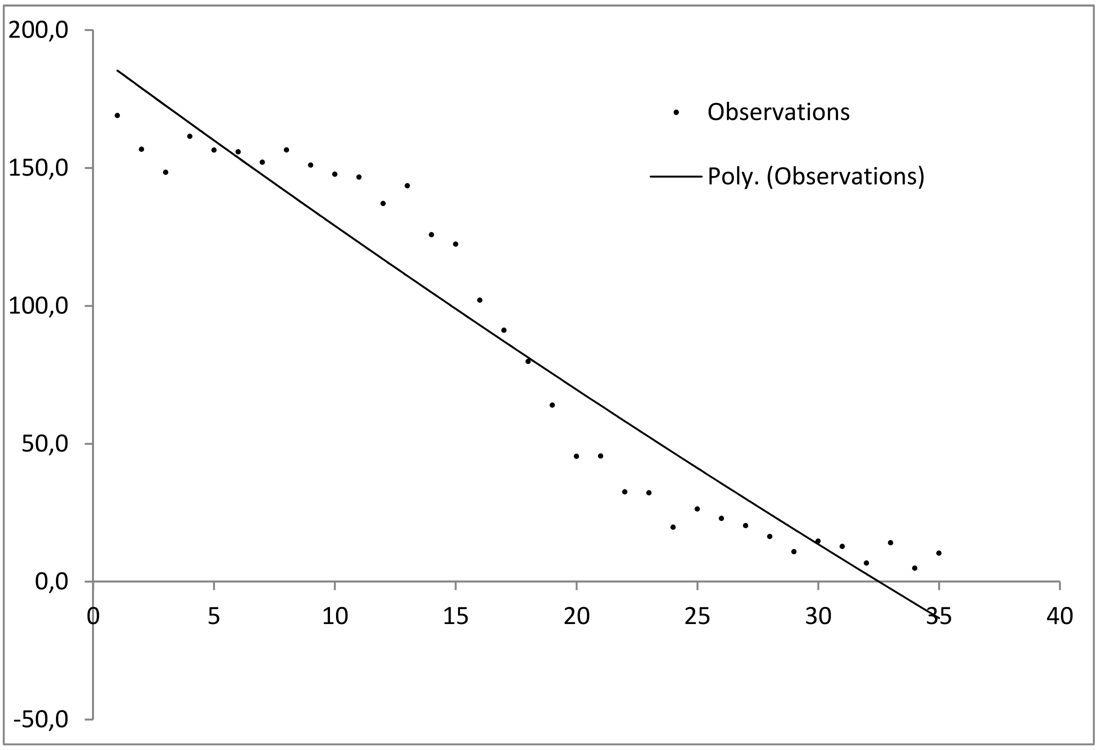

Figure 3.14. Polynomial trend line of 2nd order

Thus we have applied nonlinear polynomial regression analysis. Both second and third order polynomial regression statistics output with the critical values of *F* statistics are presented below in Figure 3.14 and Figure 3.15 and in Table 3.14 and Table 3.15 correspondingly.

Table 3.14

ANOVA - Polynomial regression of 2nd order for case 3

Source	df	SS	MS	F
Regression	2	121855,2277	60927,6138	197,6728
Residual	32	9863,1879	308,2246	
Total	34	131718,4156		

The critical value of *F* at confidence level of 95% (α = 0.05) is much lower (3.3) than the observed value of *F* (197.67). Polynomial regression of second order explains a significant proportion of the variation in the given dataset.

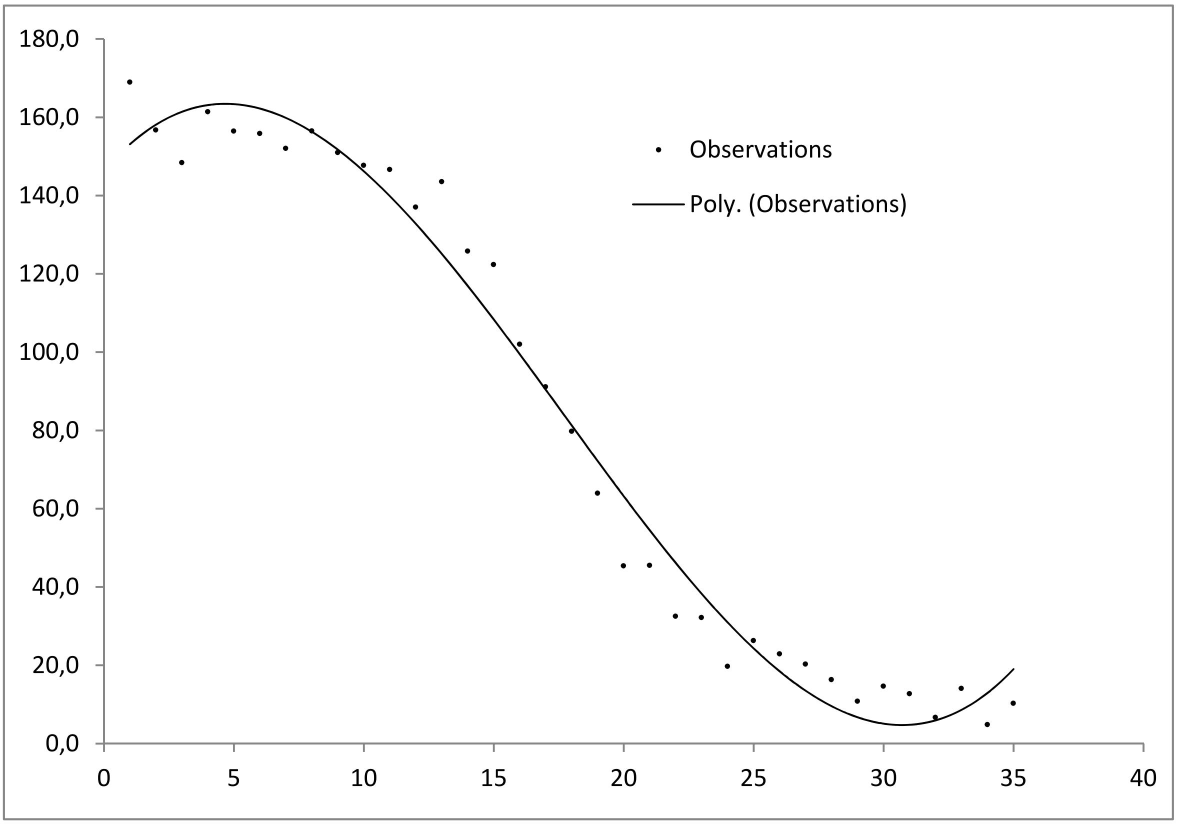

Figure 3.15. Polynomial trend line of 3rd order

Table 3.15

ANOVA - Polynomial regression of 3rd order for case 3

Source	df	SS	MS	F
Regression	3	129179,0160	43059,6720	525,6557
Residual	31	2539,3996	81,9161	
Total	34	131718,4156		

The critical value of *F* at confidence level of 95% ($\alpha = 0.05$) is much lower (2.91) than the observed value of *F* (525.66). Polynomial regression of third order explains a significant proportion of the variation in the given dataset.

3.3.3.5. Discussion of results for case 3

The regression statistics for three, five, and seven backup points used in our integrated demand model demonstrates the significance in interpreting the given polynomial dataset of observations. The *F* statistics (1375.88, 1181.70, and 902.90 correspondingly for three, five, and seven backup points) were more than their critical values (2.91, 2.55, and 2.37 correspondingly for three, five, and seven backup points). The presented regression results of the integrated demand model has no doubt validate adequacy of the mentioned regression model.

Both second and third order polynomial regression statistics presented significance in fitting the data (*F* statistics 197.67 and 525.66 with critical values of 3.3 and 2.91 correspondingly for second order and third order of polynomial regression). The third order of polynomial regression gives better interpretation of the data in means of *F* statistics. Visually the second order of polynomial regression gives a straight line whereas the third order of polynomial regression passes through the data set in the form of inverse S curve. However the integrated regression model showed better result in interpreting given data, than the polynomial regression.

The best explanation for the significant proportion of the observed dataset, from integrated demand model results, is presented by the three backup points' case (*F* value = 1375.88), followed by five backup points' case (*F* value = 1181.70), and then by seven backup points' case (*F* value = 902.90). It is easy to see that the adequacy of all three models is extremely high, so we cannot conclude, from this point of view, which model is preferable.

If there is no a priori pure economical requirements on number of backup points (for example, one have empirical economical information which lets to the necessity of using five backup points) then it is clear that model with minimal backup points should be chosen due to obvious simplicity.

3.3.4. Summing up remarks

On the cases discussed above the computational power of the developed integrated market demand model is shown. The model presented its adequacy and ability to explain significant proportion of the variation in the samples provided. Three different cases of observation datasets with different shapes has resulted in quite good *F* statistics. All calculated *F* statistics are more than their critical values from table. The choice for the quantity of backup points should be based on the rationale that there is no meaning in obtaining as high *F* statistics as possible. The examples above lead to the conclusion that three-four backup points are enough to get the results satisfying the solution of the problem.

The conventional polynomial (nonlinear) regression provides similar results; however it only provides us with trend line of the best possible explanation for the given dataset and given polynomial order. But it does not give us any information about internal structure of demand price interaction.

We suggest a new approach which is based on the assumption that nonlinear dependence between demand and price is a result of several (finite) linear demand summations. In our model these elementary demand functions are formed based on the backup points mentioned in the provided examples that are applied to determine values of demand intercepts d_i (2.9). In the direct problem case, the integrated market demand model provides the market demand function based on the elementary demand functions. Therefore, we can take a closer look at the elements that form the integrated market demand function for further analysis of price schedule, revenue and profit maximization.

Further sections of current chapter will provide samples of application of direct and inverse problem cases to the demand analysis in more details as well as the calculation of the optimal values of demand price interaction for revenue maximization.

3.4. Application of Model to Demand Curve Problems

3.4.1. Inverse problem of market demand estimation

Firstly, we are going to define smoothed market demand function from the given datasets. Secondly, based on the smoothed market demand function and assumed backup points, we are going to calculate smoothed elementary demand functions that have contributed to the smoothed market demand function. We assume, for the simplicity that smoothed elementary demand functions are of linear form.

We assume, for the sake of demonstration, that there are three backup points which are evenly distributed along the price axis. The backup points may not be distributed at even distances and may be more or less than three in amount. Three backup points assume that there are three groups of customers for the product under analysis differentiated by their income level, or geographical location, or consumption habits.

The determination of the backup points is solely the priority of the analyst who is going to apply the proposed method, and may be based on some observations of the customers' behavior for the good and service under consideration, as well as on some other observation of indicators as income distribution, economic environment, wealth, sector of the business operation, and so on. It is not, for the moment, the subject of interest.

3.4.1.1. Inverse problem of demand: Case with non-intersecting elementary demands

We assume dataset of 30 observations of demand D that form the cloud of polynomial shape. The Figure 3.16 depicts the cloud of observations, the smoothed market demand curve, and the smoothed elementary demand lines. The smoothed market demand curve has three sections as it was assumed to have three backup points. The elementary demand curves do not intersect with each other. It can be interpreted that the good or service under consideration is necessity among the all three groups of customers and have similar trend of purchasing behavior. As long as the price increases the demand decreases, customers with less income start either switching to the substitute products or cutting their consumption.

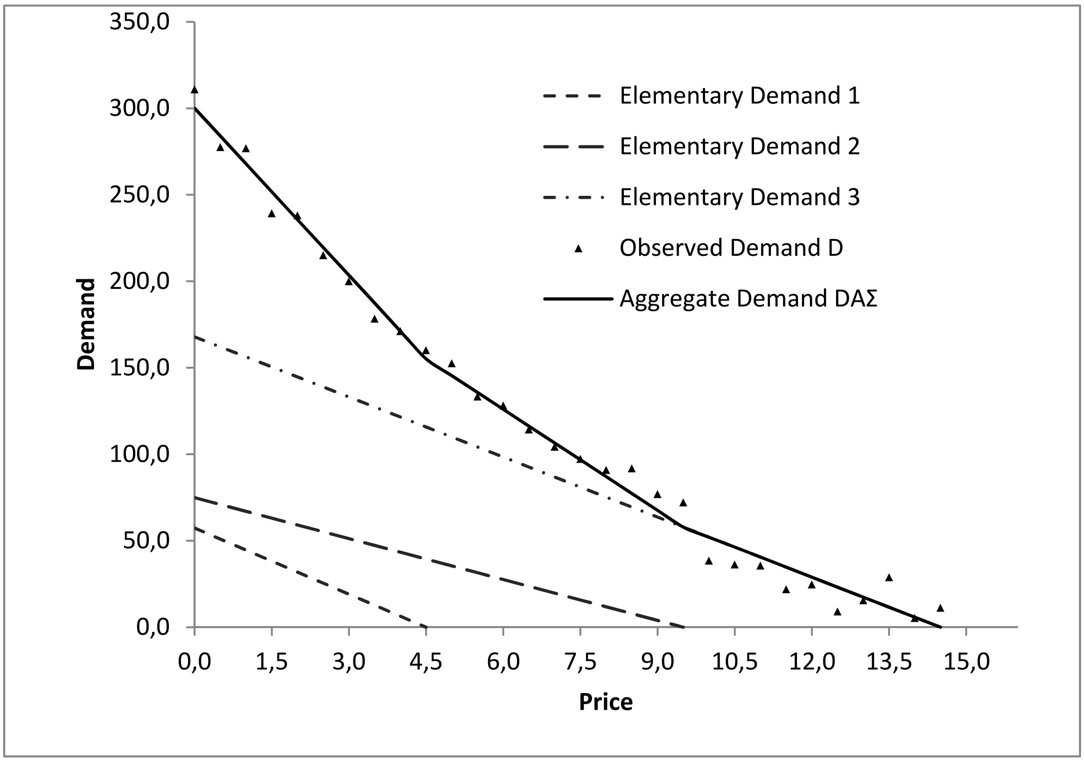

Figure 3.16. Observed demand D, smoothed elementary demands and smoothed market demand $D_{A\Sigma}$

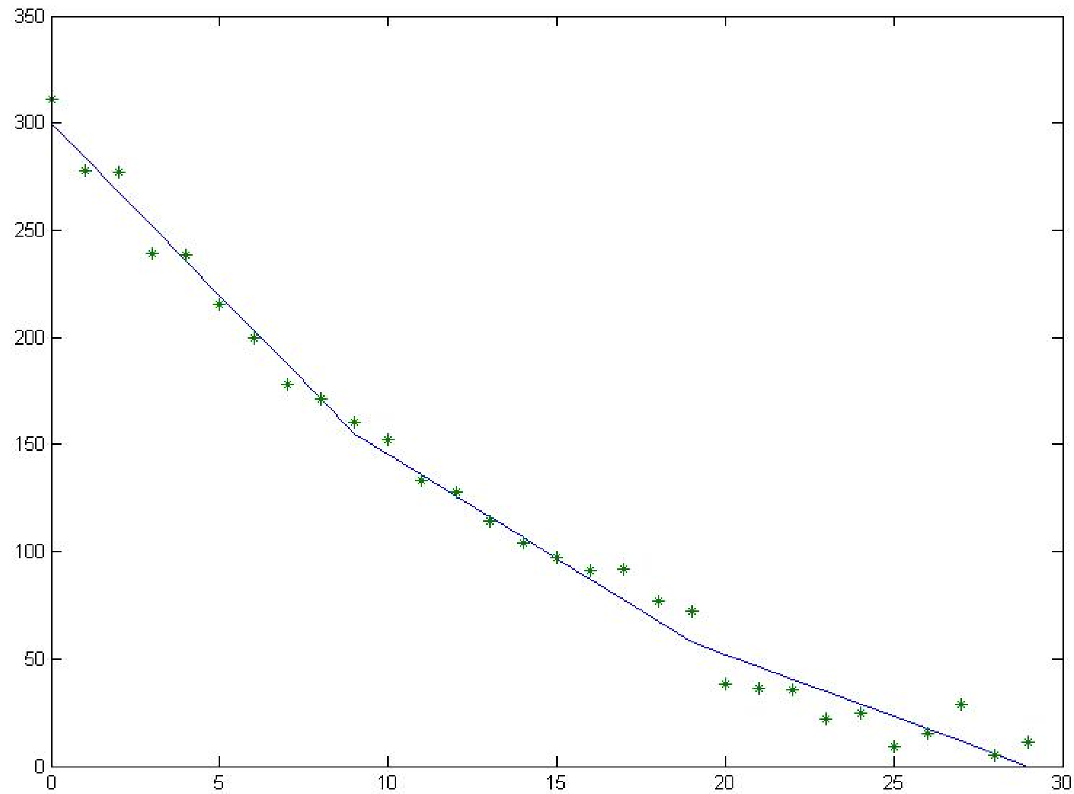

Figure 3.17. Observed demand D and smoothed market demand $D_{A\Sigma}$ (MATLAB program output)

The regression coefficients of the observed dataset are presented in Table 3.16.

Table 3.16

Regression Coefficients for inverse problem case 1

Coef	StdErr	tStat	pVal
57.33	11.992	4.7519	6.4687e-05
74.89	23.481	3.2971	0.0028292
167.85	21.373	7.7104	3.5063e-08

At the five per cent level of significance and 26 degree of freedom the critical value of t is 2.06. The calculated t-statistic for all three coefficients is more than the critical value at given significance level. This means that there is evidence of a relationship between the dependent and independent variable, because the probability of such a large t-statistic occurring if there were no relationship is very small, typically less than 0.05.

Table 3.17

ANOVA for inverse problem case 1

Source	df	SS	MS	F	P
Regr	3.0000	238653.9243	79551.3081	922.0008	0.0000
Resid	26.0000	2243.3105	86.2812		
Total	29.0000	240897.2348			

The ANOVA results of the observed dataset are presented in Table 3.17. The R^2 is equal to 0.99. The 99 per cent of the variation in demand is explained by the relationship with price. Only one per cent of the variation in demand is unexplained by the relationship, so it can be omitted. This means that the estimates of the regression coefficients are reliable.

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (2.98) than the observed value of F (922.0), which means that integrated regression model with three backup points adequately describes the given data by the provided means of measurement. In other words, the regression model with three backup points does explain a significant proportion of the variation in the sample. It is therefore highly unlikely that such sample data would be observed if there was no relationship between the variables involved.

Table 3.18

Estimated Parameters of Elementary Demands for inverse problem case 1

Parameters	Elementary Demand 1	Elementary Demand 2	Elementary Demand 3
Slope	-12.74	-7.89	-11.58
Intercepts	57.33	74.89	167.85

Applying the mentioned method we have determined the demand intercepts for the assumed backup points, as well as we have obtained the slopes of the three smoothed elementary demands. The estimated parameters of the smoothed elementary demand curves are presented in Table 3.18. Therefore if the price were zero the group of customers with less income would purchase 57.33 amounts of the product, the group of customers with medium level of income would purchase 74.89 amounts of the product, and the group of the customers with the high level of income would purchase 167.85 amounts of the product.

3.4.1.2. Inverse problem of demand: Case with intersecting smoothed elementary demands

We assume dataset of 21 observations of demand *D* that form the cloud of polynomial shape. The Figure 3.18 depicts the cloud of observations, the smoothed market demand curve, and the smoothed elementary demand lines. The smoothed market demand curve has three sections as it was assumed to have three backup points. The two of the elementary demand curves do intersect with each other. The group of customers with low income have steeper smoothed elementary demand line and low level of purchases than that of both medium and high income level smoothed elementary demand lines. The medium income level smoothed elementary demand line is steeper than the high income level smoothed elementary demand line. These can be interpreted that the good or service under consideration is not necessity for the low income level group of customers, so that demand is not high. As long as the price increases the demand decreases, customers with less income start either switching to the substitute products or cutting their consumption.

The high income level group of customers are less sensitive to the changes in price, whereas other two groups are quite sensitive to the changes in price level.

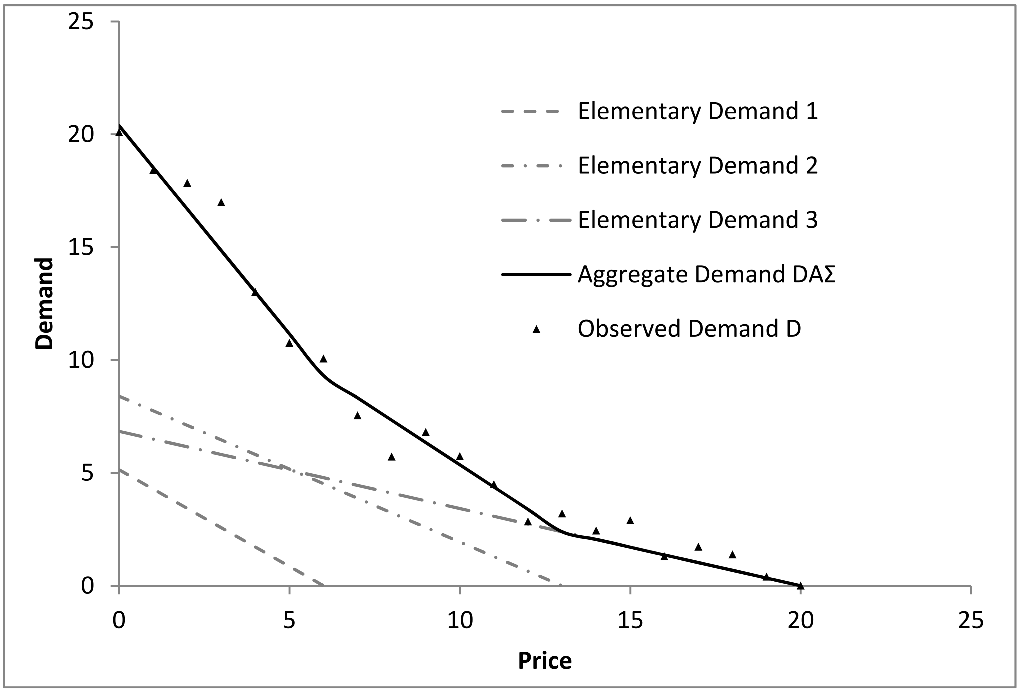

Figure 3.18. Observed demand D, smoothed elementary demands and smoothed market demand $D_{A\Sigma}$

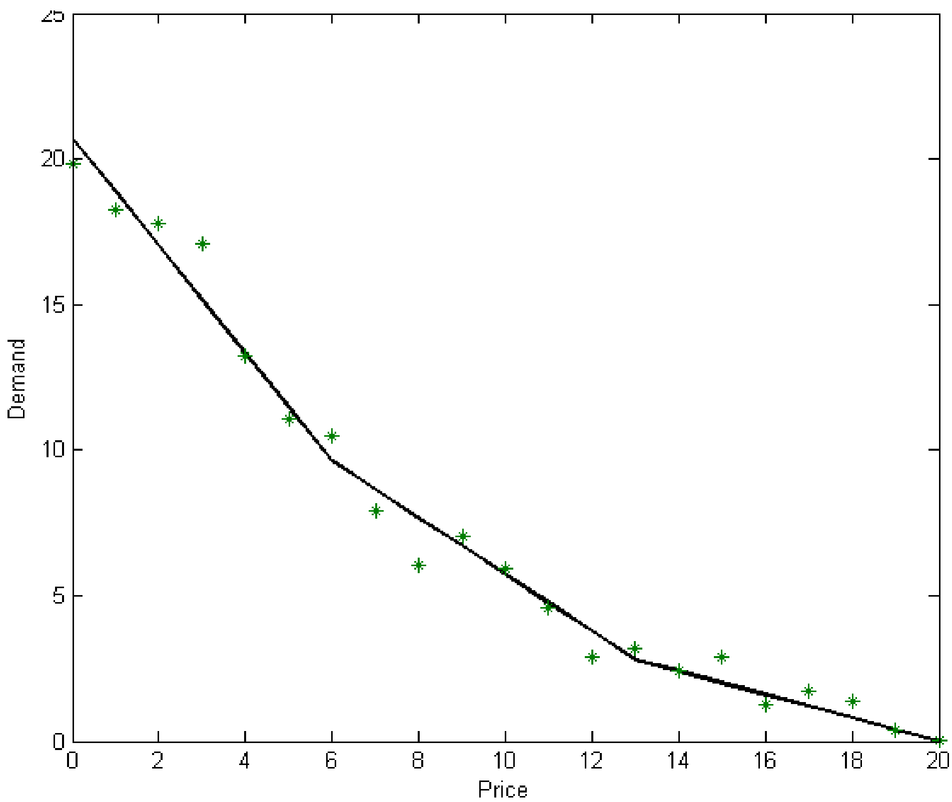

Figure 3.19. Observed demand D and smoothed market demand $D_{A\Sigma}$ (MATLAB program output)

The population of high income level customer group is high so that the demand is high, or the product under consideration is some type of luxury so that only customers with enough income can afford it even at quite high price levels.

The regression coefficients of the observed dataset are presented in Table 3.19.

Table 3.19

Regression Coefficients for inverse problem case 2

Coef	**StdErr**	**tStat**	**pVal**
5.1422	1.2491	4.1166	0.0007204
8.399	2.4298	3.4567	0.0030141
6.8343	2.2076	3.0958	0.0065633

At the five per cent level of significance and 17 degree of freedom the critical value of *t* is 2.11. The calculated *t*-statistic for all three coefficients is more than the critical value at given significance level. This means that there is evidence of a relationship between the dependent and independent variable, because the probability of such a large *t*-statistic occurring if there were no relationship is very small, typically less than 0.05.

Table 3.20

ANOVA for inverse problem case 2

Source	**df**	**SS**	**MS**	**F**	**P**
Regr	3.0000	824.9669	274.9890	390.8435	0.0000
Resid	17.0000	11.9608	0.7036		
Total	20.0000	836.9277			

The ANOVA results of the observed dataset are presented in Table 3.20. The R^2 is equal to 0.99. The 99 per cent of the variation in demand is explained by the relationship with price. Only one per cent of the variation in demand is unexplained by the relationship, so it can be omitted. This means that the estimates of the regression coefficients are reliable.

The critical value of F at confidence level of 95% ($\alpha = 0.05$) is much lower (3.20) than the observed value of F (390.84), which means that integrated regression model with three backup points adequately describes the given data by the provided means of measurement. In other words, the regression model with three backup points does explain a significant proportion of the variation in the sample. It is therefore highly unlikely that such sample data would be observed if there was no relationship between the variables involved.

Table 3.21

Estimated Parameters of Elementary Demands of inverse problem case 2

Parameters	Elementary Demand 1	Elementary Demand 2	Elementary Demand 3
Slope	-0.867	-0.65	-0.342
Intercepts	5.14	8.4	6.84

Applying the mentioned method we have determined the demand intercepts for the assumed backup points, as well as we have obtained the slopes of the three smoothed elementary demands. The estimated parameters of the smoothed elementary demand curves are presented in Table 3.21. Therefore if the price were zero the group of customers with less income would purchase 5.14 amounts of the product, the group of customers with medium level of income would purchase 8.4 amounts of the product, and the group of the customers with the high level of income would purchase 6.84 amounts of the product.

3.4.2. Direct problem of market demand estimation

The direct problem of market demand, as it was mentioned in the theoretical section of the study, summarizes to defining the smoothed market demand D_s from elementary observed demands D_i. Firstly, we are going to obtain smoothed elementary demands D_{si} from the elementary observed demands D_i, independently from each other, by application of linear regression method. We assume linear regression equations for the elementary observed demands D_i. Secondly, we are going to substitute coefficient values of linear regression equations of smoothed elementary demands into developed model (2.9) to obtain smoothed market demand D_s.

We assume three observations of elementary demand D_i: elementary observed demand D_1 sample of 6 observations, elementary observed demand D_2 sample of 11 observations, and elementary observed demand D_3 sample of 16 observations. We assume, for the simplicity, that smoothed elementary demand functions are of linear form. The graphical representations of elementary observed demands along with the linear regression coefficients are presented in Figure 3.20, Figure 3.21, and Figure 3.22.

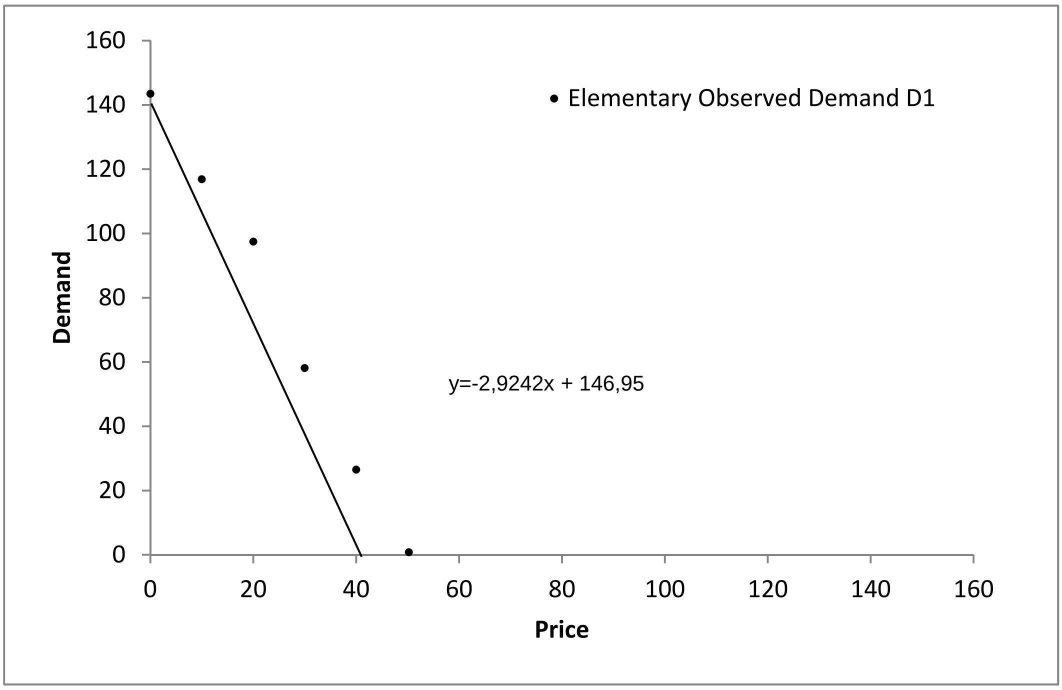

Figure 3.20. Elementary observed demand D_1 and linear regression coefficients

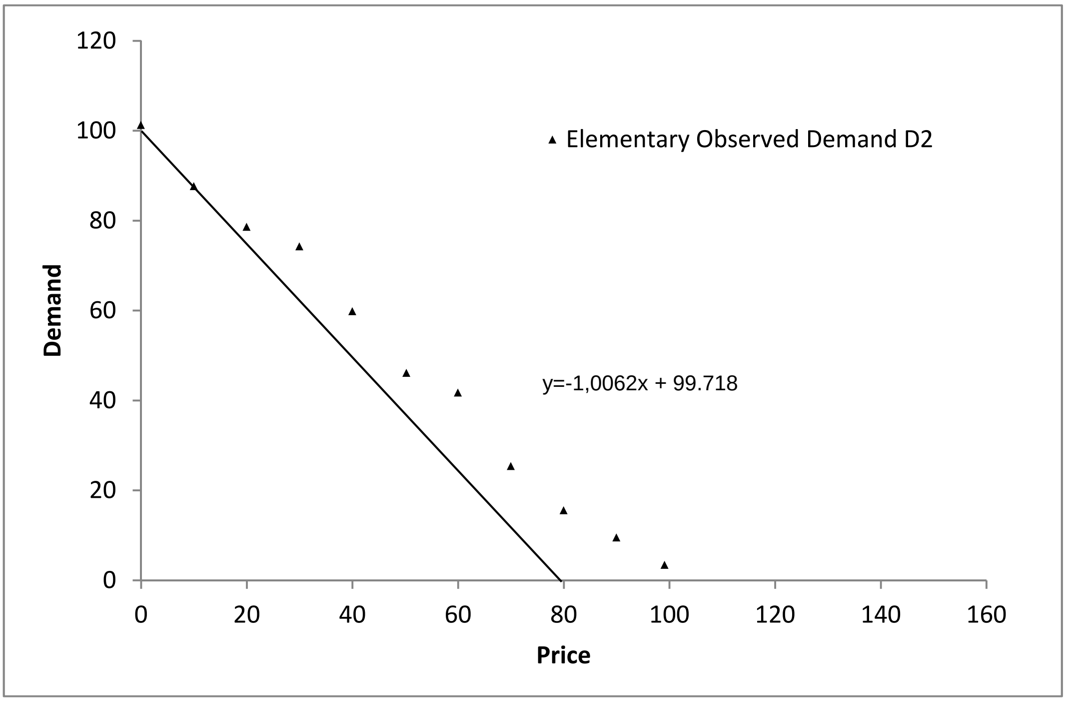

Figure 3.21. Elementary observed demand D_2 and linear regression coefficients

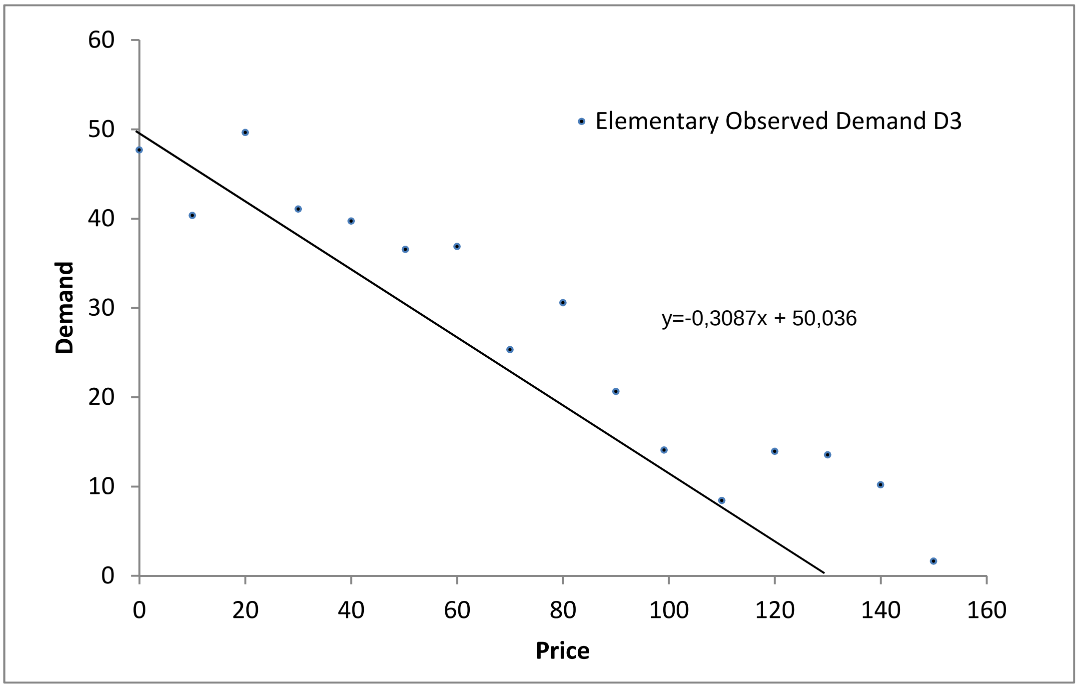

Figure 3.22. Elementary observed demand D_3 and linear regression coefficients

The summary of the estimated parameters for smoothed elementary demands D_{si} are presented in Table 3.22.

Table 3.22

Estimated Parameters of Smoothed Elementary Demands of direct problem

Parameters	Elementary Demand 1	Elementary Demand 2	Elementary Demand 3
Slope	-2.9242	-1.0062	-0.3087
Intercepts	146.95	99.718	50.036

In order to obtain backup points, the linear regression equations are set equal to zero and solved for x. The calculated backup points are summarized in Table 3.23.

Table 3.23			
Calculated Backup Points(x_i) for Smoothed Elementary Demands of direct problem			
Parameters	**Elementary Demand 1**	**Elementary Demand 2**	**Elementary Demand 3**
Regression Equations	y= -2.9242 * x + 146.95	y= -1.0062 * x + 99.718	y= -0.3087 * x + 50.036
Backup Points	50.253	99.104	162.09

The intercept (d_i) and backup point (x_i) parameters are substituted to the model (2.9) to calculate parameters of smoothed market demand D_s. The results of calculation are summarized in Table 3.24.

Table 3.24				
Calculated Parameters of Smoothed Market Demand D_s of direct problem				
Backup points	0	50.253	99.104	162.09
Aggregated demand amounts	296.70	83.68	19.44	0.0

If price for the product under consideration is set to zero, then the market demand equals to 296.7. If price is set to 50.253 then the market demand is 83, and when the price is set to 99.104 the market demand equals to 19.44. At the price 162.09 there is no any quantity of the product is demanded, so the market demand is equals to zero.

The result of the application of the estimated parameters to the model (2.9) is graphically represented in Figure 3.23.

The datasets of observed elementary demands are summed using model (2.9) so that at each corresponding price (z) the sum of the observations are obtained. The cloud of integrated observations is obtained, which is nicely estimated in figure 3.23 by the smoothed integrated demand D_s.

Further section will discuss the cases of optimal values of demand – price interactions for maximization of the total revenue.

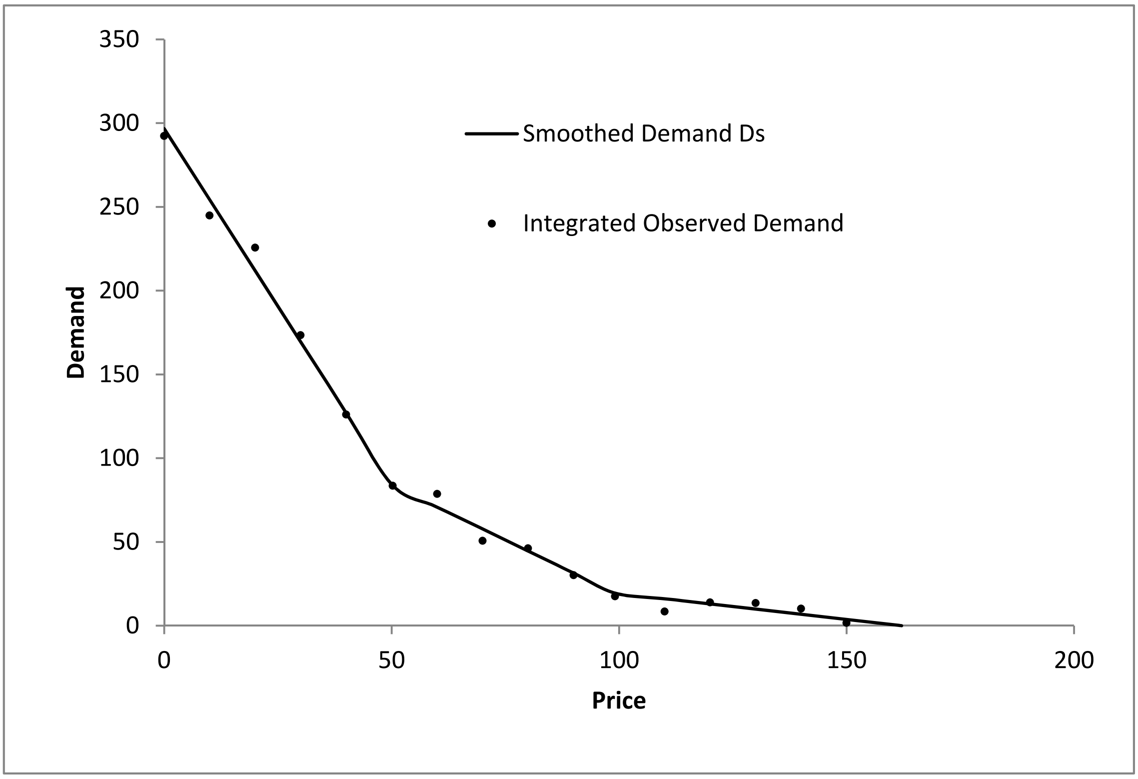

Figure 3.23. Integrated observed demand and smoothed integrated demand D_s

3.5. Optimal Values of Integrated Demand Model

The criterion (2.62) puts forward three positions for the value of integrated optimal revenue: explicit, implicit, and adjacent. Let us discuss each case separately.

Each case assumes three elementary demand functions that contribute to the integrated demand function, that is $r = 3$. The backup points x_i are known beforehand. The unitless price z is simply the ration of x_i values divided by the highest x_i value, and to get the corresponding real value of price one should multiply the z value with the highest x_i value. The application of model (2.9) to the dataset of market demand observations estimates the intercepts d_i. The integrated total revenue curve is calculated by substituting x_i and d_i values to the total revenue equation (2.51). The total revenue maximization values are obtained by solving (2.54) for given x_i and d_i values, and total revenue maximization prices are calculated by (2.53).

3.5.1. One maximum: one explicit maximum, and two implicit maxima of integrated total revenue curve

Let's assume following demand and price interaction for the elementary demands as in Table 3.25. The given values are the intercepts at corresponding axes of Price and Quantity. The data means that at given price values the zero quantity of product demanded, and at given demand quantities the price values are equals to zero. The unitless price indicates prices in the scale from 0 to 1 for calculation purposes.

Table 3.25

Demand and Price Schedule

Elementary Demands	**1**	**2**	**3**
Price (x_i)	70	100	150
Unitless Price (z)	0.47	0.67	1
Demand (d_i)	150	100	50

Table 3.26 summarizes the results of the calculations of TRx_i, z_{op_k}, and *TRmax* at each backup points by the formulas indicated in the introduction of the 3.4 section. The calculations themselves are not provided due to that that anyone with adequate calculus capabilities can come up to these obvious outcomes.

Table 3.26

Estimated Optimal Values at each backup points

Backup Points	**1**	**2**	**3**
TRx_i	26,4	11,1	0
z_{op_k}	0.29	0.38	0.5
TRmax	43.15	28.13	12.50

The Figure 3.24 depicts visual representations of above mentioned calculated results. The solid line presents the integrated total revenue curve as a result of the interaction of the total revenues generated by each elementary demand lines presented visually on Figure 3.25 and numerically on Table 3.25. The labels *A*, *B*, and *C* are representing corresponding maxima total revenues (*TRmax*) as a result of summation of revenues of elementary demand lines. By label *A* is depicted the maximum point of integrated total revenue curve.

It is a maximum of the integrated total revenue curve and explicitly defined. The labels *B* and *C* are implicitly defined maxima. They are placed under the integrated total revenue curve, and the results of interaction of revenue curves of second and third, and only third elementary demand lines correspondingly. The square dotted vertical lines labeled TRx_i are presenting the backup points of the elementary demand lines.

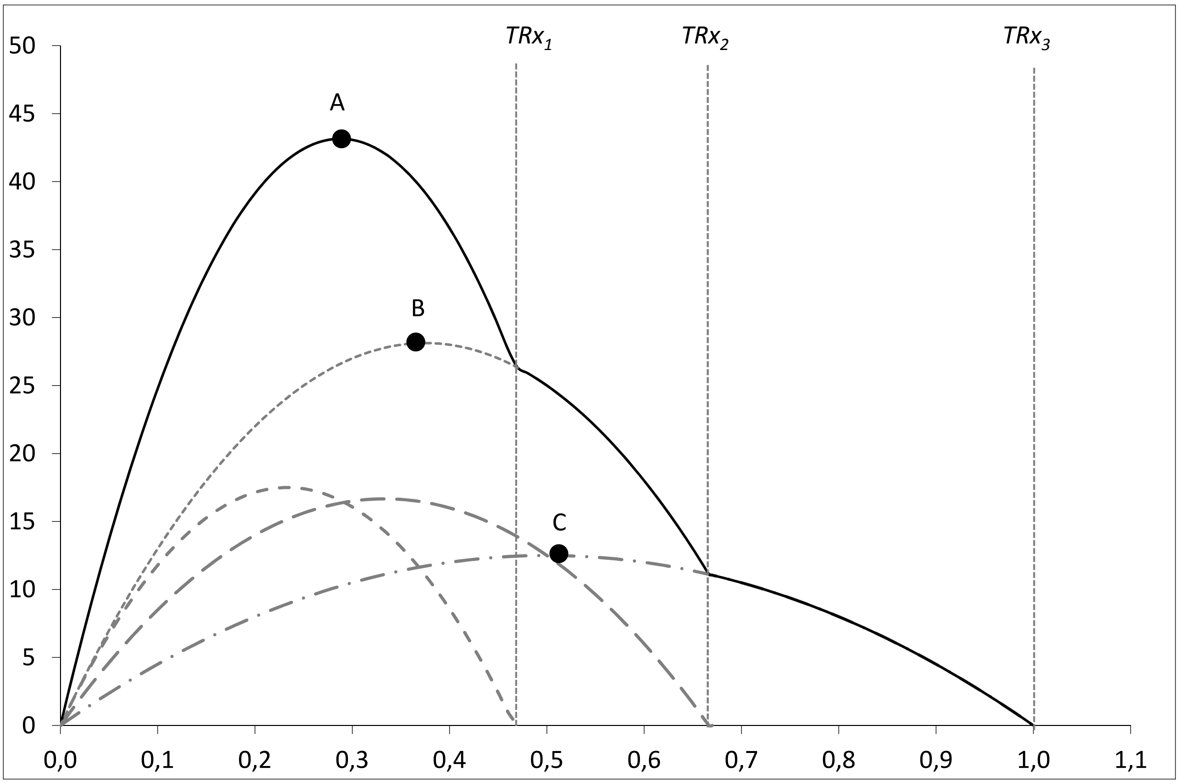

Figure 3.24. Integrated total revenue curve, one explicit and two implicit total revenue maxima:

- *Solid curve*: integrated total revenue, it is the summation of revenues of three elementary demand lines;
- *Square dotted line*: summation of revenues of Elementary demand 2 and Elementary demand 3 lines;
- *Dash-dotted parabola:* revenue curve of Elementary demand 3 line;
- *Long dashed parabola:* revenue curve of Elementary demand 2 line;
- *Dashed parabola:* revenue curve of Elementary demand 1 line;
- *Point A*: explicit maximum of integrated total revenue;
- *Points B and C*: implicit maxima of integrated total revenue;
- *Square dotted vertical lines:* backup points of the elementary demand lines

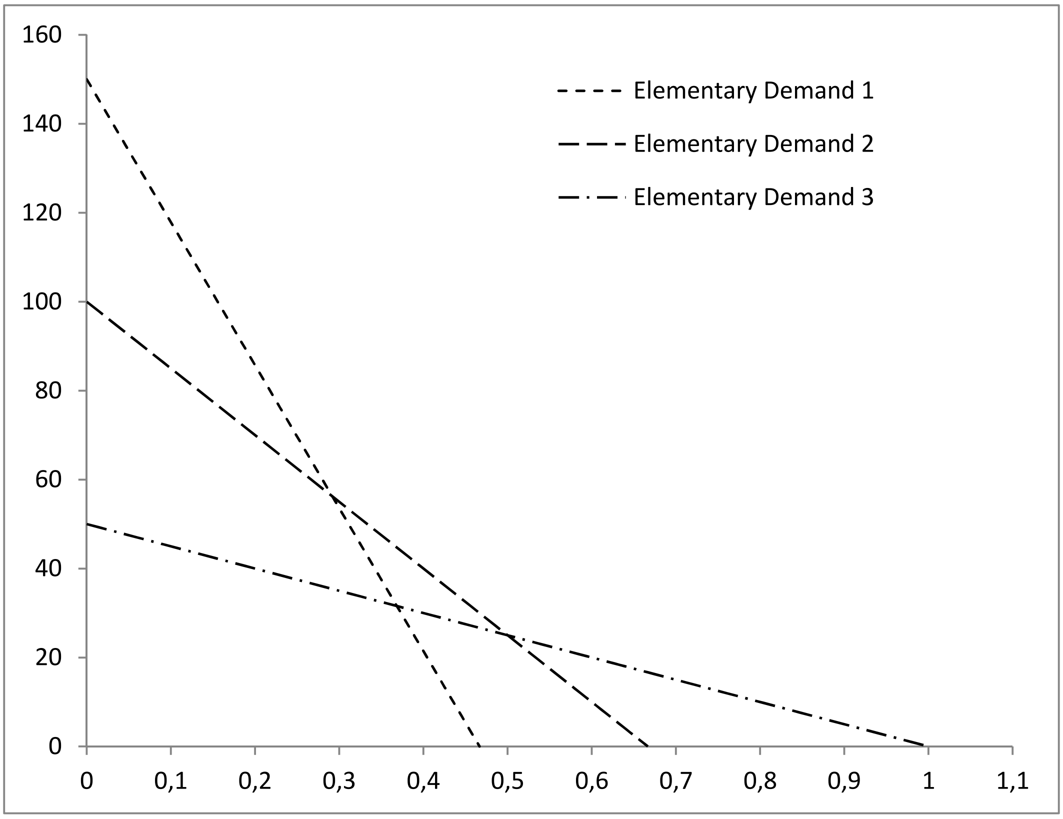

Figure 3.25. Elementary demand lines

Definitely it is clear that in a case with these observed elementary demands we have only one total revenue maximization point, which is obviously and no doubt is the maximum for given curve. The integrated total revenue curve has continuous shape with no minimums, and hidden two maxima.

3.5.2. Several explicit maxima of integrated total revenue curve

The previous case introduced one maximum point of integrated total revenue curve, and some interesting insight to the minimums of the total revenue curve in controversy to conventional studies. It is clear that the distinction of the current study from others is that the total revenue curve is obtained from piecewise differentiable elementary demand lines.

It is obvious and clear that solving for that maximization point will yield to the business entity maximum of the revenues. However, the insights into total revenue curve may not be limited only to the previously discussed cased.

Let us introduce a case with several explicitly defined maxima of total revenue curve from possible variations of shapes of total revenue curve.

Let's assume following demand and price interaction for the elementary demands as in Table 3.27. The given values are the intercepts at corresponding axes of Price and Quantity. The data means that at given price values the zero quantity of product demanded, and at given demand quantities the price values are equals to zero. The unitless price indicates prices in the scale from 0 to 1 for calculation purposes.

Table 3.27

Demand and Price Schedule

Elementary Demands	**1**	**2**	**3**
Price (x_i)	30	45	115
Unitless Price (z)	0.26	0.39	1
Demand (d_i)	100	125	150

Table 3.28 summarizes the results of the calculations of TRx_i, z_{op_k}, and $TRmax$ at each backup points by the formulas indicated in the introduction of the 3.4 section. The calculations themselves are not provided due to that that anyone with adequate calculus capabilities can come up to these obvious outcomes.

Table 3.28

Estimated Optimal Values at each backup points

Backup Points	**1**	**2**	**3**
TRx_i	39.8	35.7	0
z_{op_k}	0.22	0.29	0.5
TRmax	41.23	40.27	37.50

The Figure 3.26. Integrated total revenue curve and several explicit maximdepicts visual representations of above mentioned calculated results. The solid line presents the integrated total revenue curve as a result of the interaction of the total revenues generated by each elementary demand lines presented visually on Figure 3.27 and numerically on Table 3.28. The labels *A*, *B*, and *C* are representing corresponding maxima total revenues (*TRmax*) as a result of summation of revenues of elementary demand lines. By label *A* is depicted the maximum point of integrated total revenue curve. It is a maximum of the integrated total revenue curve and explicitly defined. The labels *B* and *C* are also explicitly defined maxima. They are placed on the integrated total revenue curve, and they are the results of interaction of revenue curves of second and third, and only third elementary demand lines correspondingly. The square dotted vertical lines labeled TRx_i are presenting the backup points of the elementary demand lines.

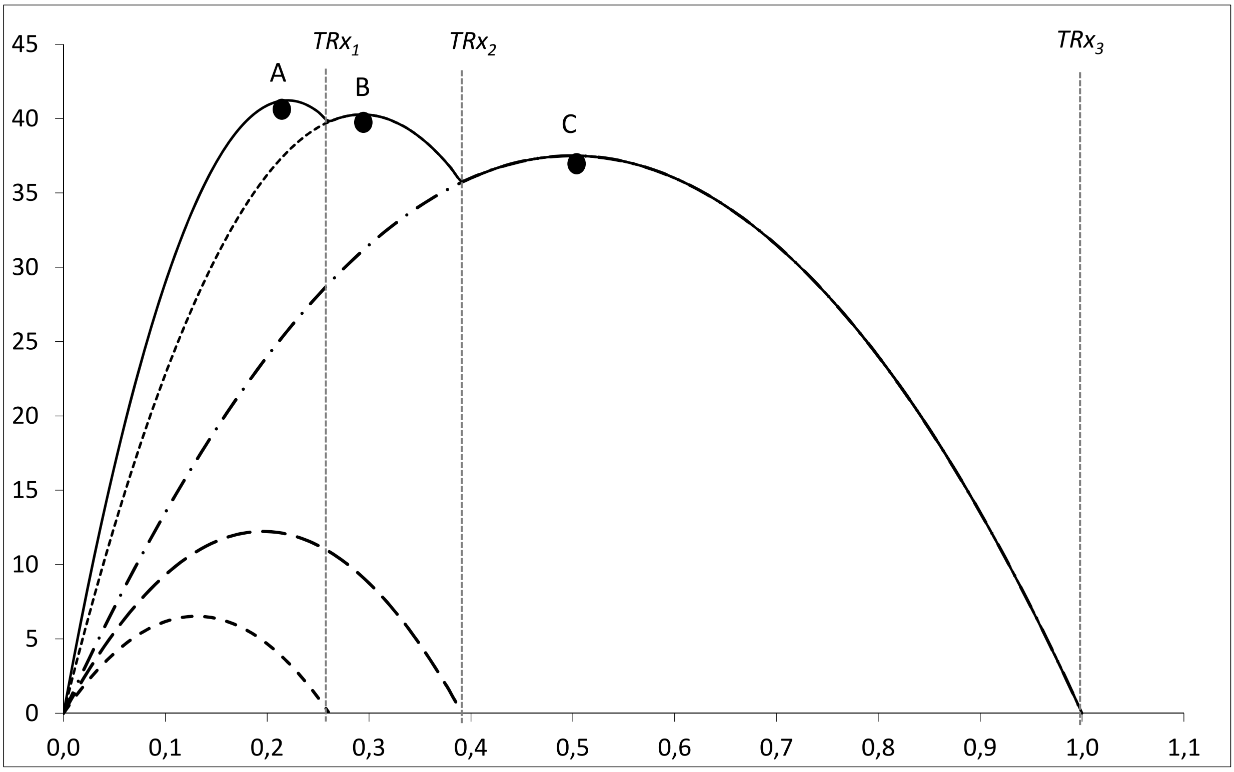

Figure 3.26. Integrated total revenue curve and several explicit maxima:

- *Solid curve*: integrated total revenue, it is the summation of revenues of three demand lines;
- *Square dotted line*: summation of revenues of Elementary demand 2 and Elementary demand 3 lines;
- *Dash-dotted parabola:* revenue curve of Elementary demand 3 line;
- *Long dashed parabola:* revenue curve of Elementary demand 2 line;
- *Dashed parabola:* revenue curve of Elementary demand 1 line;
- *Point A, B, and C*: explicit maxima of integrated total revenue;
- *Square dotted vertical lines:* backup points of the elementary demand lines.

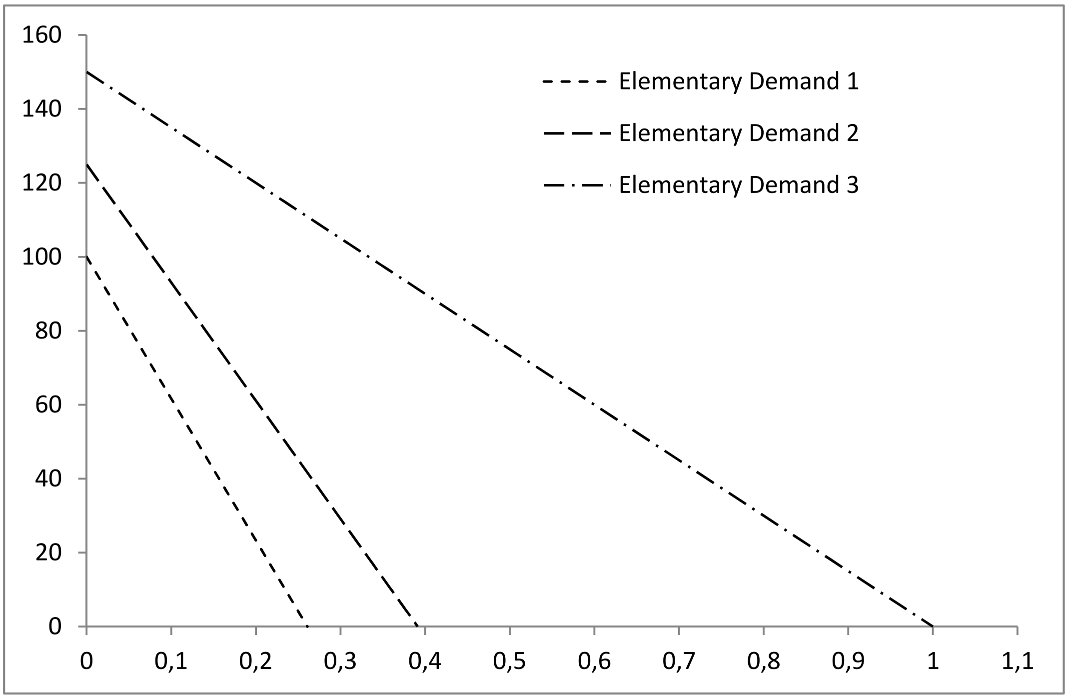

Figure 3.27. Elementary demand lines

Definitely it is clear that in the case with these observed elementary demands we have three total revenue maximization points, which are obvious and no doubt explicitly defined maxima for given curve. The integrated total revenue curve has continuous shape with no minimums, and tree maxima.

It is visually difficult to indicate the maxima of the integrated total revenue curve from the Figure 3.26, however the analytical solution for this case will definitely indicate that point labeled by capital letter *A* is the maximum point.

It is obvious that the maximum point on the curve will depend on the elementary demand lines data. If the third elementary demand line will be more in quantity demanded or the price is quite more than provided in the example, then the point labeled by the capital letter *C* will be the maximum point for the curve. The same outcome can be obtained for the point labeled *B* by increasing maximum quantity demanded or by increasing maximum price value.

3.5.3. One explicit maximum and two maxima on intersection of adjacent parabolas of the integrated total revenue curve

The previous two cases introduced integrated total revenue curve examples with one explicit maximum point and two implicit maxima points, and three explicit maximum points.

Let us introduce a new case with one explicitly defined maximum point and two maxima points on the intersection of adjacent parabolas of the integrated total revenue curve.

Let's assume following demand and price interaction for the elementary demands as in Table 3.29. The given values are the intercepts at corresponding axes of Price and Quantity. The data means that at given price values the zero quantity of product demanded, and at given demand quantities the price values are equals to zero. The unitless price indicates prices in the scale from 0 to 1 for calculation purposes.

Table 3.29

Demand and Price Schedule

Elementary Demands	**1**	**2**	**3**
Price (x_i)	50	70	140
Unitless Price (z)	0.36	0.50	1
Demand (d_i)	100	150	200

Table 3.30 summarizes the results of the calculations of *TRx*i, z_{op_k}, and *TRmax* at each backup points by the formulas indicated in the introduction of the 3.4 section. The calculations themselves are not provided due to that that anyone with adequate calculus capabilities can come up to these obvious outcomes.

Table 3.30

Estimated Optimal Values at each backup points

Backup Points	**1**	**2**	**3**
$\boldsymbol{TRx_i}$	61.20	50	0
$\boldsymbol{z_{op_k}}$	0.29	0.35	0.5
TRmax	64.90	61.25	50

The Figure 3.28 depicts visual representations of above mentioned calculated results. The solid line presents the integrated total revenue curve as a result of the interaction of the total revenues generated by each elementary demand lines presented visually on Figure 3.29 and numerically on Table 3.29. The labels *A*, *B*, and *C* are representing corresponding maxima total revenues (*TRmax*) as a result of summation of revenues of elementary demand lines. By label *A* is depicted the maximum point of integrated total revenue curve. It is a maximum of the integrated total revenue curve and explicitly defined. The labels *B* and *C* are maxima located at the intersection of adjacent parabolas of integrated total revenue curve. They are placed on the integrated total revenue curve, and they are the results of interaction of revenue curves of both second and third and third elementary demand lines correspondingly. The square dotted vertical lines labeled TRx_i are presenting the backup points of the elementary demand lines.

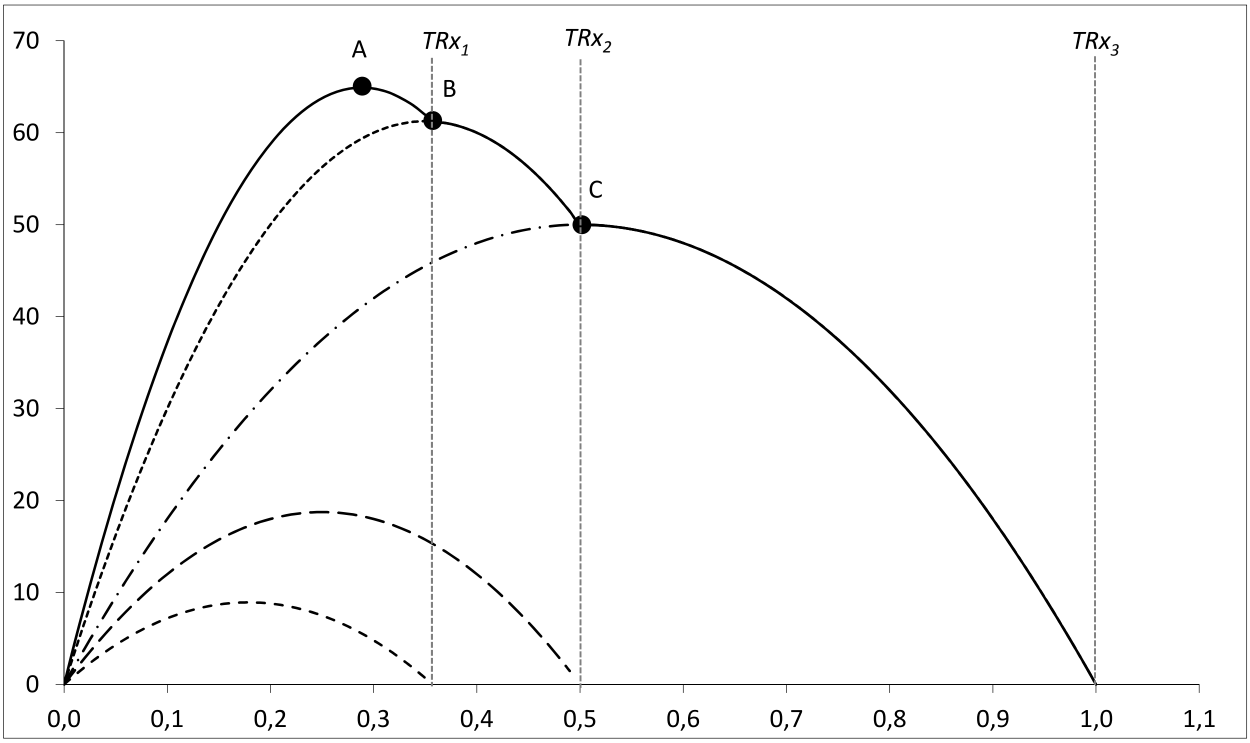

Figure 3.28. One explicit maximum and two maxima on intersection of adjacent parabolas of the integrated total revenue curve:

- *Solid curve*: integrated total revenue, it is the summation of revenues of three demand lines;
- *Square dotted line*: summation of revenues of Elementary demand 2 and Elementary demand 3 lines;
- *Dash-dotted parabola:* revenue curve of Elementary demand 3 line;
- *Long dashed parabola:* revenue curve of Elementary demand 2 line;
- *Dashed parabola:* revenue curve of Elementary demand 1 line;
- *Point A:* explicit maximum of integrated total revenue;
- *Points B and C*: maxima points on intersection of adjacent parabolas;
- *Square dotted vertical lines:* backup points of the elementary demand lines.

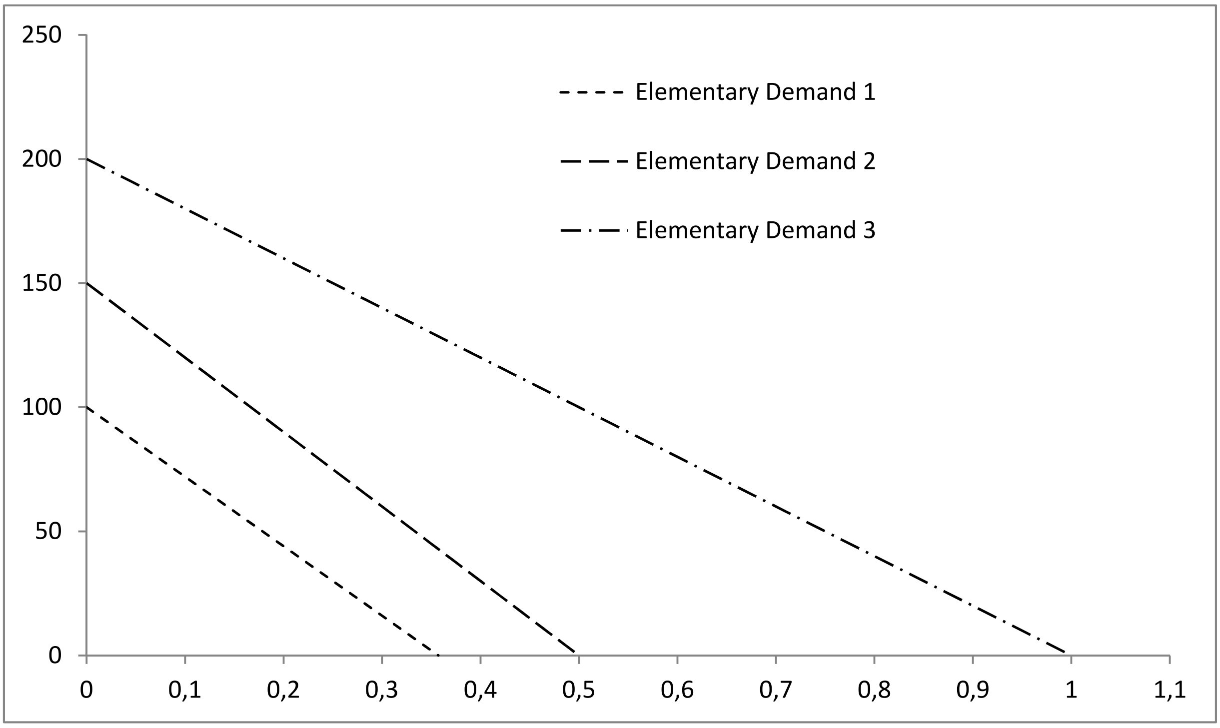

Figure 3.29. Elementary demand lines

The above example clearly depicts the case with one explicitly defined maximum point at label *A* and two maxima labeled by *B* and *C* that fall on the intersection points of adjacent parabolas of integrated total revenue curve. It is interesting to note that the total integrated revenue curve has no obvious minima; we can say that points of maxima labeled by *B* and *C* are the minima for the given integrated total revenue curve.

CONCLUSION

(Results of the research)

The following, in principal new, results, concerning nature of essentially nonlinear demand-price relationships, were obtained:

1. The observed demand was represented as the integrated, essentially non-linear, object consisted of a set of linear demand-price components;
2. On the base of the new introduced concepts of *Observed Demand, Smoothed Demand, Elementary Observed Demand, Smoothed Elementary Demands, Integrated Demand, and Integrated Total Revenue Curve*, the discrete and continuous theoretical models of integrated demands were elaborated;
3. Based on the usage of the dummy multidimensional regression technique, the new statistical method of estimation of parameters for integrated demand discrete model was elaborated. The capability of the model was demonstrated throughout several numerical examples;
4. The new integrated demand discrete model results were compared to the conventional polynomial regression analysis results. It was shown that the integrated demand discrete model gives statistically better results in explaining the observational data;
5. Appropriate software tools (in MATLAB programming language) were created;
6. Based on elaborated integrated demand discrete model the new approach to the estimation of optimal microeconomic parameters was elaborated;
7. The new methodology of determining maximums of integrated total revenue curves was elaborated;
8. Taking into consideration nonlinear specific nature of integrated demand, the new expressions for estimation of optimal values of prices, demand quantities and maximums of total revenues were obtained;
9. The new conditions and relevant criteria for existing points of maxima of the integrated total revenue curve were defined;
10. The outcomes mentioned above allowed creating the new principles of analysis and calculation of optimal micro-economical parameters for nonlinear integrated demand-price functional relationships.

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APPENDICES

Appendix A. MATLAB Program for estimations of parameters of Integrated Market Demand Model

```
function dummy(y1, n1, m1)

global f;
x=0;
xi=0;
d=0;
X=0;

n=n1; % number of points x
for i=1:n
    xi(i)=i-1;% input of array x
end

m=m1;% interpolation step
r=rem(n,m);
d(1)=x(1);

if r==0
    dl=n/m;
    for i=1:dl
        d(i+1)=xi(m*i);
        for j=1:m*i
            x(i,j)= xi(j);
            X(i,j)=(1-xi(j)/d(i+1));
         end;

     end;
  end;

  if r~=0
```

```
    dl=(n-r)/m;
      d(dl+2)=xi(n)
      for i=1:dl
         d(i+1)=xi(m*i);
          for j=1:m*i
            x(i,j)= xi(j);
            X(i,j)=(1-xi(j)/d(i+1));
           end;
       end;
     for j=1:n
        x(dl+1,j)=xi(j);
        X(dl+1,j)=(1-xi(j)/d(dl+2));
     end;
  end;
 Xt = X';

function [regcoef1]=regress(y1,Xt)

  stats = regstats(y1,Xt);
    %_________________block_of
Output_of_regression______________
    t = stats.tstat;

    regcoef1=t.beta;

    %______________block_of_output_of_regression_and correlation

    r=stats.adjrsquare;

    Table_of_Reg_Coeff =
dataset({t.beta,'Coef'},{t.se,'StdErr'},
{t.t,'tStat'},{t.pval,'pVal'})
```

```
   %______________block_of_output_of_ANOVA___________________
    f=stats.fstat;
    fprintf('\n')
    fprintf('Table_of_ANOVA');
    fprintf('\n\n')
    fprintf('%6s','Source');
    fprintf('%10s','df','SS','MS','F','P');
    fprintf('\n')
    fprintf('%6s','Regr');
    fprintf('%10.4f',f.dfr,f.ssr,f.ssr/f.dfr,f.f,f.pval);
    fprintf('\n')
    fprintf('%6s','Resid');
    fprintf('%10.4f',f.dfe,f.sse,f.sse/f.dfe);
    fprintf('\n')
    fprintf('%6s','Total');
    fprintf('%10.4f',f.dfe+f.dfr,f.sse+f.ssr);
    fprintf('\n')

    f=regcoef1(1:3)'*Xt';

 end  %of functions

regcoef=regress(y1,Xt);

XI=X*y1;
XC=X*X';
p=inv(XC)*XI;
y2=y1';
f1=p'*X;
figure;
plot(xi(1:n),f1(1:n),xi(1:n),y2(1:n));

end
```